Maximum LPV-SS realization in a static form

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I. Foreword

This note is about an augmented state-space realization of LPV input-output forms with static dependency. The importance of this form is due to the fact that it only requires static dependence and it is always realizable. Next step: apply model reduction on it to get to the minimal LPV-SS form with only static dependency.

II. Realization

Consider an LPV-IO representation in the form of

$$y(k)+\sum_{i=1}^{n_\mathrm{a}} A_i(p_k)q^{-i}y(k)=\sum_{j=0}^{n_\mathrm{b}} B_j(p_k)q^{-j}u(k), \tag{1}$$

where $n_\mathrm{a}, n_\mathrm{b}\geq 0$, $y:\mathbb{Z}\rightarrow\mathbb{R}^{n_\mathrm{y}}$ is the output, $u:\mathbb{Z}\rightarrow\mathbb{R}^{n_\mathrm{u}}$ is the input and $p:\mathbb{Z}\rightarrow\mathbb{R}^{n_\mathrm{p}}$ is the scheduling signal respectively, while $A_i:\mathbb{R}^{n_\mathrm{p}}\rightarrow\mathbb{R}^{n_\mathrm{y}\times n_\mathrm{y}}$ and $B_i:\mathbb{R}^{n_\mathrm{p}}\rightarrow\mathbb{R}^{n_\mathrm{y}\times n_\mathrm{u}}$ are given matrix functions. The the maximum augmented state-space realization of (1) reads as

$$\left[\begin{array}{c|c} A(p_k) & B(p_k) \\ \hline C(p_k) & D(p_k)\end{array}\right]=$$

$$\left[\begin{array}{cccccccc|c}
-A_1(p_k) & \cdots & -A_{n_\mathrm{a}-1}(p_k) & -A_{n_\mathrm{a}}(p_k) & B_1(p_k) & \cdots & B_{n_\mathrm{b}-1}(p_k) & B_{n_\mathrm{b}}(p_k) & B_0(p_k) \\
I & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\
0 & \cdots & I & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & I \\
0 & \cdots & 0 & 0 & I & 0 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & I & 0 & 0 \\ \hline
-A_1(p_k) & \cdots & -A_{n_\mathrm{a}-1}(p_k) & -A_{n_\mathrm{a}}(p_k) & B_1(p_k) & \cdots & B_{n_\mathrm{b}-1}(p_k) & B_{n_\mathrm{b}}(p_k) & B_0(p_k)
\end{array}\right].$$

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