Data-driven LPV modeling of continuous pulp digesters

TUE-CS-2013-002

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May 1, 2013

Abstract

In this technical report, the LPV-IO identification techniques described in Kauven et al. [2013] (Chapter 5) are applied in order to estimate an LPV model of a continuous pulp digester. The pulp digester simulator (described in Modén [2011]) has been provided by ABB for benchmark studies as part of its participation in the EU project Autoprofit.

1   Description of the continuous pulp digester

The continuous pulp digester is an upright standing tubular reactor (see Fig. 1) where wood chips together with *withe liquor* (a strongly aqueous solution of sodium hydroxine and sodium sulfide) are heated under pressure to dissolve the lignin in the wood chips and thus to obtain almost pure cellulose fibers (wood pulp). These fibers are used to make all kinds of paper products.

Wood chips and white liquor are fed continuously at the top, and the chips travel downwards slowly, surrounded by free liquor and full of entrapped liquor in their porosities. Additional liquor, a mixture of white and wash liquor, is added at some points along the digester, and *black liquor* (spent mixture of white liquor and lignin) is extracted at a few points as well. The required heat for the conversion is provided as latent heat by the feed liquor flows. The temperatures of the liquor flows can be controlled. In the bottom of the reactor, pulp and liquor leave the digester. Depending on the zone of the reactor, the resulting flow of free liquor is counter-current or co-current with the chip flow direction.

The quality of the pulp is indicated by the amount of remaining lignin in the pulp. A measure for this is the Kappa number. The smaller the Kappa number the higher is the quality of the pulp. The quality is positively influenced by a longer resting (cooking) time of the pulp, higher temperatures and larger liquor flows. It is worth remarking that in the process, though unwanted, some of the cellulose are dissolved, causing lower yield. Therefore, there is always a trade-off due to the fact that if the cook process is longer or the temperature is higher, more lignin is removed, but, at the same time, more cellulose is lost.
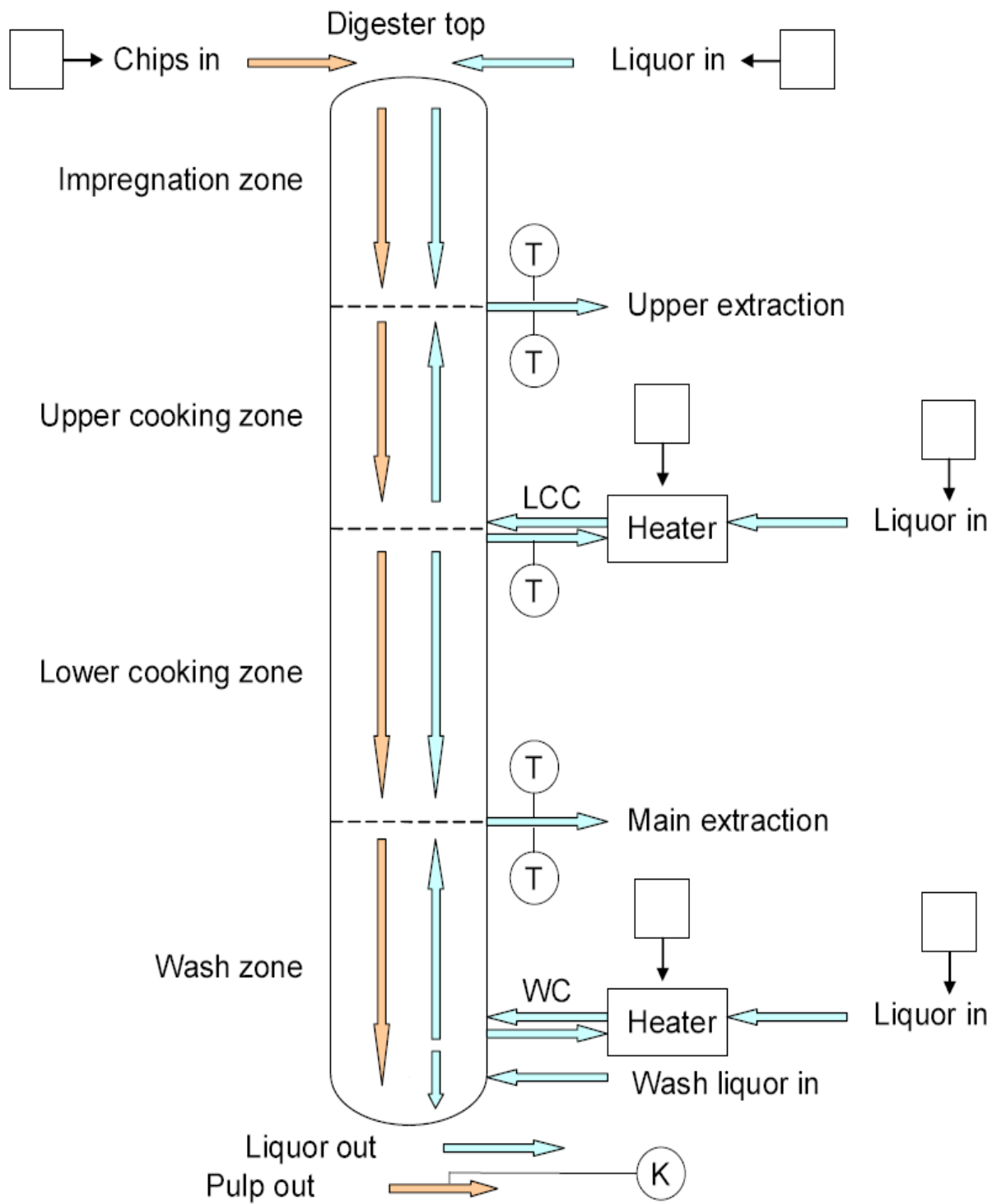
Chips in

Digester top

Liquor in

Impregnation zone

T

Upper extraction

T

Upper cooking zone

LCC

Heater

Liquor in

T

Lower cooking zone

T

Main extraction

T

Wash zone

WC

Heater

Liquor in

Wash liquor in

Liquor out

Pulp out

K

Figure 1: A schematic view of the pulp digester.

2 Simulator description

The basic core of the model implemented in the simulator provided by ABB is the extended Purdue model introduced in Wisnewski et al. [1997], which consists of a set of nonlinear *ordinary differential equations* (ODEs) modeling the three phases of the process: the solid phase of the wood chips, the entrapped-liquor phase, and the free-liquor phase. Exchange between the phases is also considered.

The *manipulated variables* (controller variables used to influence the process) are:

MV1 Temperature setpoint to LCC (lower cook circulation) heater;

MV2 Temperature setpoint to WC (wash circulation) heater;

MV3 Alkali-to-wood ratio with liquor feed to digester top;

MV4 Alkali-to-wood ratio with liquor feed to LCC;

MV5 Alkali-to-wood ratio with liquor feed to WC.

The *disturbance variable* (which is assumed to be measured) is:

D1 Chip feed rate.

The *process variables* (outputs of the process model) are:

PV1 Kappa number,

PV2 Liquor temperature in extraction from impregnation zone;

PV3 Liquor temperature in extraction from upper cooking zone;

PV4 Liquor temperature in extraction from lower cooking zone;

PV5 Liquor temperature in extraction from washing zone;

PV6 Liquor temperature in recirculation to LCC heater.

3 Simulation scenarios

The pulp digester is simulated for two different species of wood chips (hardwood and softwood) and for two different set-points for the Kappa number (Kappa number = 90 and Kappa number = 30). Then, the four different operating conditions considered in simulation are:

- Hardwood/Kappa number= 90;

- Softwood/Kappa number = 90;

- Hardwood/Kappa number = 30;

- Softwood/Kappa number $= 30$.

Details about the compositions of hardwood and softwood are available in Modén [2011].

A closed-loop simulation is performed. More precisely, a linear MPC has been designed based on the linearization of the Purdue model around the operating point. The considered (quadratic) cost function penalizes the variations of all the manipulated variables and the deviations of the Kappa number w.r.t. the setpoint. The temperature setpoints (i.e., MV1 and MV2) are constrained to be smaller than 170 °C and the alkali-to-wood ratios (i.e., MV3, MV4 and MV5) are not allowed to be negative.

For each operating condition, the data are gathered by simulating the behavior of the pulp digester for about 14 days (corresponding to 2000 samples measured with a time interval of 10 minutes). The chip feed rate is a (measurable) stochastic variable, normally distributed with a mean of 0.025 m^3/s and a standard deviation of 0.005 m^3/s. The measurements of the Kappa number are corrupted by an additive Gaussian white noise $e_o(t)$ with zero mean and standard deviation 0.03. This corresponds to a *signal-to-noise ratio* (SNR) equal to about 8 db, where the SNR is defined as

$$
\text{SNR} = 10 \log_{10} \left(\frac{\sum_{t=1}^{2000} \left(w_o(t) - \bar{K} \right)^2}{\sum_{t=1}^{2000} e_o^2(t)} \right), \tag{1}
$$

where $w_o(t)$ is the noise-free measurement of the Kappa number at time t, while $\bar{K}$ is either 90 or 30, depending on the considered operating condition.

The available data are splitted into two sets, i.e. an estimation data set $\mathcal{D}_N$ (which consists of the first $N = 1500$ data and is used to estimate a model of the pulp digester) and a validation data set $\mathcal{D}_V$ (which consists of the last $N_V = 500$ data and is used to test the performance of the estimated model).

4 LPV model structure

The pulp digester dynamics are described by a MISO LPV model with:

- Kappa number (i.e., PV1) as output;

- MV1, MV2, MV3, MV4 and MV5 as inputs;

- Chip feed rate (i.e., DV1) and liquor temperature in extraction from impregnation zone (i.e., PV2) as scheduling parameters.

The LPV model structure has been selected based on physical insights on the system behavior and through a trial-and-error procedure, which aimed at maximizing the *best fit rate* (BFR) w.r.t. the validation data set $\mathcal{D}_V$, where the BFR is defined as

$$
\text{BFR} = \max\{0, 1 - \frac{\|y - \hat{y}\|_2}{\|y - \bar{y}\|_2}\}, \tag{2}
$$

with y denoting the measured output (i.e., Kappa number), $\hat{y}$ the estimate of Kappa number, and $\overline{y}$ the sample mean of the (noise-corrupted) system output y in the validation data set. As physical insight for the choice of the regressors we used the residence time of the pulp in the different zones of the pulp digester (cf. Modén [2011]). Hence, a certain number of recent inputs and outputs (different from the Kappa number) should not enter the regression vector. Using the BFR as criterion, the trial-and-error procedure led to a model where the value of the Kappa number at time t depends on:

- Kappa number at time $t - 1$ and $t - 2$;

- MV1 at time $t - 12$, $t - 13$ and $t - 14$;

- MV2 at time t, $t - 1$ and $t - 2$;

- MV3 at time $t - 12$, $t - 13$ and $t - 14$;

- MV4 at time $t - 32$, $t - 33$ and $t - 34$;

- MV5 at time t, $t - 1$ and $t - 2$;

- DV1 at time t, $t - 1$ and $t - 2$;

- PV2 at time $t - 20$.

5 Obtained results

For each considered operating condition (i.e., Hardwood/Kappa number = 90; Softwood/Kappa number = 90; Hardwood/Kappa number = 30; Softwood/Kappa number = 30), an LPV model of the pulp digester is estimated through the identification algorithms described in Kauven et al. [2013]. It is important to point out that, when the parametric LPV identification approaches are applied, an LPV model with affine dependencies on the scheduling variables is assumed.

In case of nonparametric LPV identification, *Radial Basis Functions* (RBFs) are used as kernels $K(\bullet, \bullet)$, i.e.,

$$K(p(j), p(k)) = \exp\left(\frac{(p(j) - p(k))^2}{\sigma^2}\right), \tag{3a}$$

where $p(k)$ is the value of the scheduling parameter vector at time k and $\sigma > 0$ is an hyperparameter chosen by the user to control the width of the RBF. The value of σ is chosen through a cross-validation procedure, that is by maximizing (through an exhaustive search) the BFR (eq. (2)) w.r.t the validation data set $\mathcal{D}_V$. Of course, different values of σ are obtained in each of the considered operating conditions.

For the sake of comparison, an LTI model of the pulp digester is also estimated.

First, the Hardwood/Kappa number = 90 operating condition is considered. The obtained results are reported in Table 1, which shows the performance of the estimated models in terms of BFR and MSE w.r.t. the validation data set $\mathcal{D}_V$, where the MSE is given by

$$\text{MSE} = \frac{1}{N} \sum_{t=1}^{N_V} (y(t) - \hat{y}(t))^2. \tag{4}$$

Table 1 also shows the computational time required by Matlab to estimate the model on a 2.40-GHz Intel Pentium IV with 3 GB of RAM. The outputs of the identified models over the validation set are plotted in Figs. 2(a)-7(a), together with the true values of Kappa number. Figs. 2(b)-7(b) show the differences between the estimates and the true values of Kappa number.

The other three operating conditions are then considered. The performance (in terms of BFR, MSE and required computational time) of the estimated models are summarized in Table 2, Table 3 and Table 4. Only the outputs of the LPV models estimated through the open-loop parametric LPV identification approach are plotted (Figs. 8-10). It is important to remind that an LPV model is identified at each operating condition.

The obtained results show that, besides having a low computational complexity, the parametric LPV identification algorithms achieved better performance in modeling the behavior of the pulp digester in comparison to the LS-SVM based approaches. Furthermore, although the data are gathered in a closed-loop setting, the performance achieved by the open-loop identification schemes are comparable, and sometimes even better, than the ones achieved by the instrumental-variable based algorithms. It is also important to point out that the estimated LPV models described more accurately the nonlinear dynamics of the pulp digester compared to LTI models. Therefore, the LPV modeling paradigm seems to be an interesting approach for improving the performance of the closed-loop system.

References

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P. E. Modén. Continuous pulp digester simulation model. *ABB document: SECRC/AT/TR-2011/035*, 2011.

P. A. Wisnewski, F. J. Doyle III, and F. Kayihan. Fundamental continuous-pulp-digester model for simulation and control. *AIChE journal*, 43(12):3175–3192, 1997.

Table 1: **Hardwood/Kappa number = 90**. Best fit rate (BFR), mean square error (MSE) and required computational time (in seconds).

	Parametric LPV (open-loop)	Parametric LPV (closed-loop)	Nonparametric LPV (open-loop)	Nonparametric LPV (closed-loop)	LTI (open-loop)	LTI (closed-loop)
BFR	88%	90%	81%	82%	22%	21%
MSE	0.0030	0.0023	0.0072	0.0067	0.1318	0.1366
time	1.5	12	620	2432	0.8	10

Table 2: **Softwood/Kappa number = 90**. Best fit rate (BFR), mean square error (MSE) and required computational time (in seconds).

	Parametric LPV (open-loop)	Parametric LPV (closed-loop)	Nonparametric LPV (open-loop)	Nonparametric LPV (closed-loop)	LTI (open-loop)	LTI (closed-loop)
BFR	82%	78%	70%	70%	24%	23%
MSE	0.0231	0.0327	0.0578	0.0579	0.3806	0.3859
time	1.0	17	665	2521	0.8	9

Table 3: **Hardwood/Kappa number = 30**. Best fit rate (BFR), mean square error (MSE) and required computational time (in seconds).

	Parametric LPV (open-loop)	Parametric LPV (closed-loop)	Nonparametric LPV (open-loop)	Nonparametric LPV (closed-loop)	LTI (open-loop)	LTI (closed-loop)
BFR	83%	84%	70%	66%	23%	21%
MSE	0.0036	0.0033	0.0105	0.0132	0.0701	0.0738
time	1.5	12	601	2285	0.9	9

Table 4: **Softwood/Kappa number = 30**. Best fit rate (BFR), mean square error (MSE) and required computational time (in seconds).

	Parametric LPV (open-loop)	Parametric LPV (closed-loop)	Nonparametric LPV (open-loop)	Nonparametric LPV (closed-loop)	LTI (open-loop)	LTI (closed-loop)
BFR	74%	72%	56%	55%	28%	28%
MSE	0.0274	0.0341	0.0769	0.0792	0.1966	0.1965
time	1.4	15	687	2505	0.8	11

(a) Kappa number   (b) Error
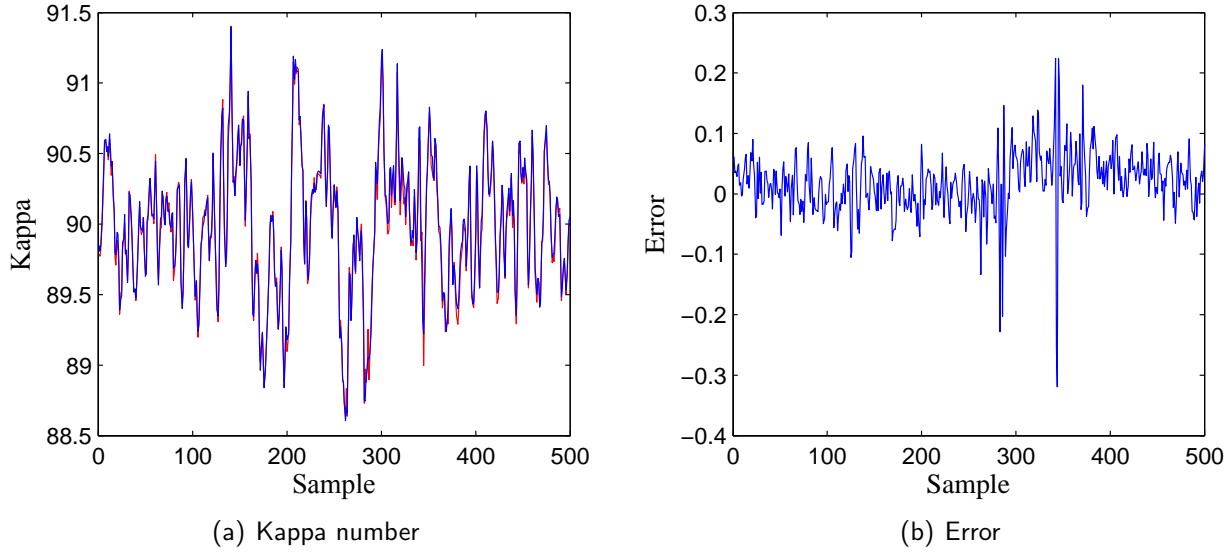
Figure 2: Hardwood/Kappa number = 90; **Parametric LPV** identification (**open-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.
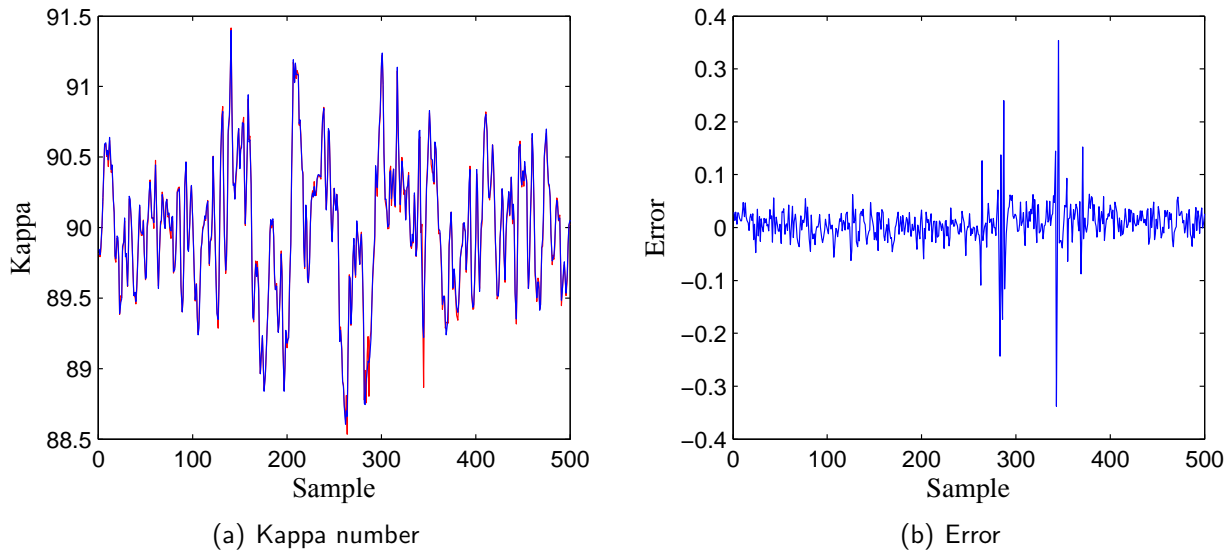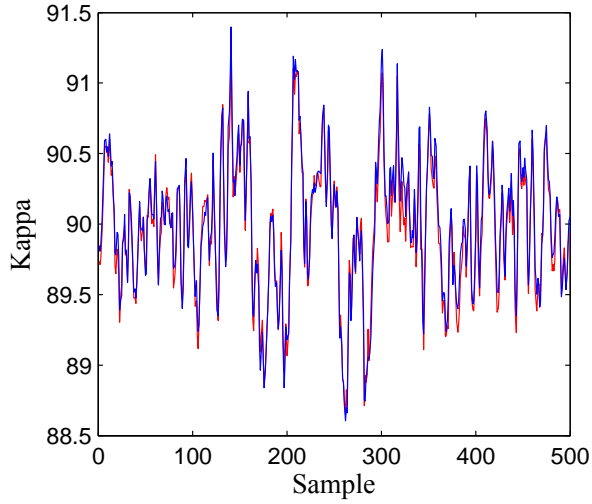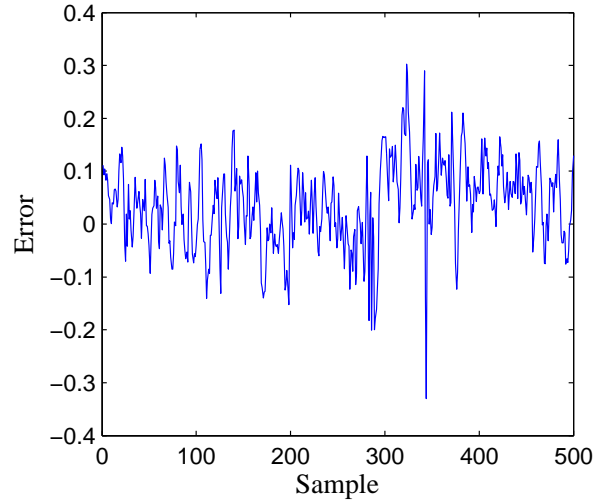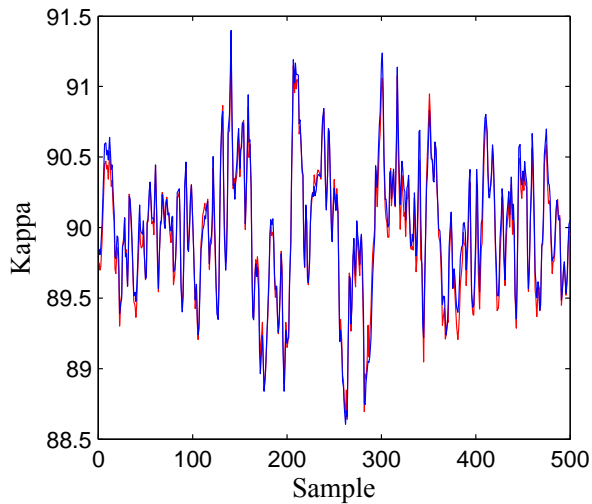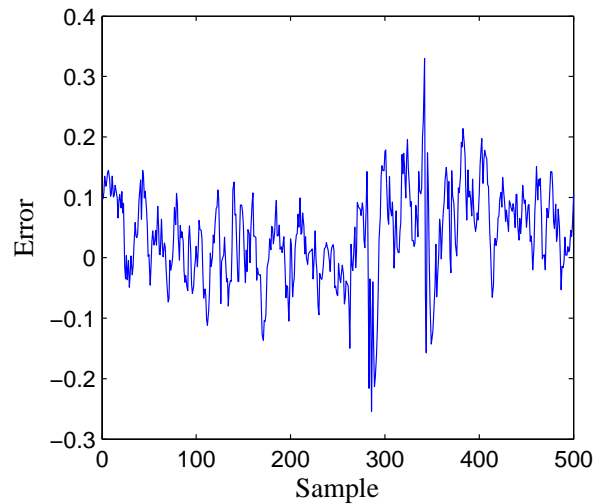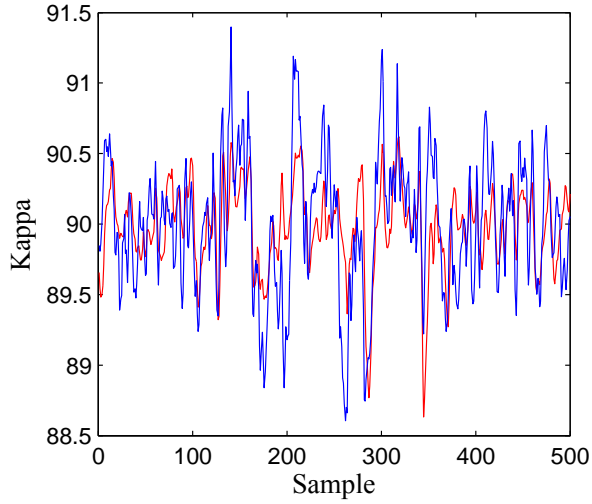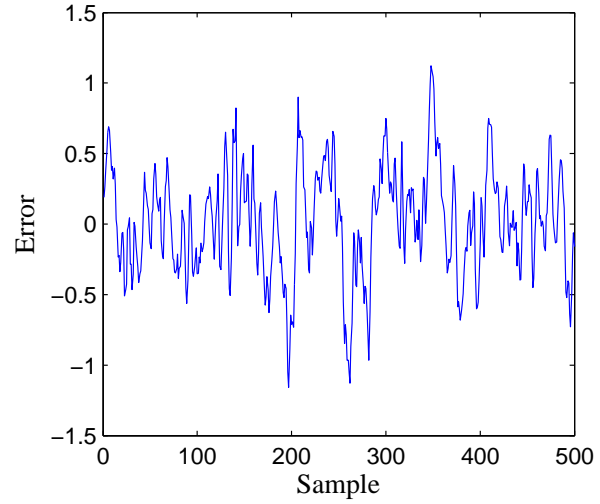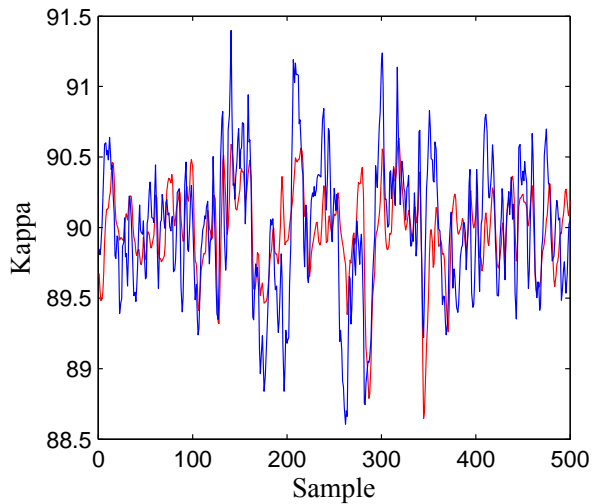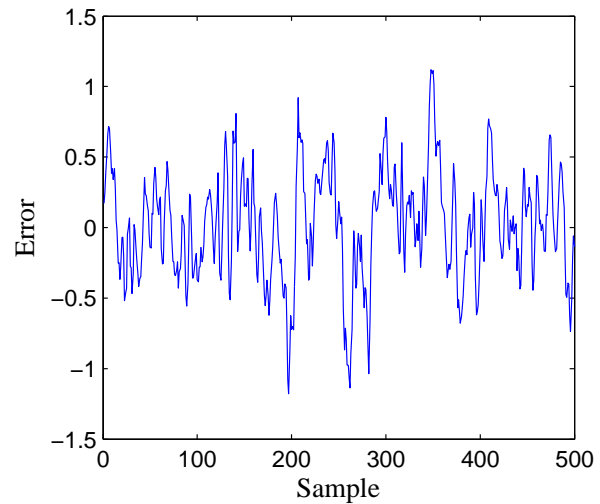

(a) Kappa number   (b) Error

Figure 3: Hardwood/Kappa number = 90; **Parametric LPV** identification (**closed-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.

| (a) Kappa number | (b) Error |

Figure 4: Hardwood/Kappa number = 90; **Nonparametric LPV** identification (**open-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.
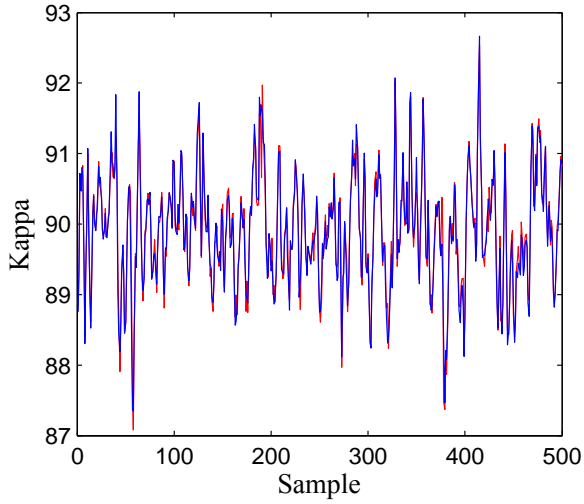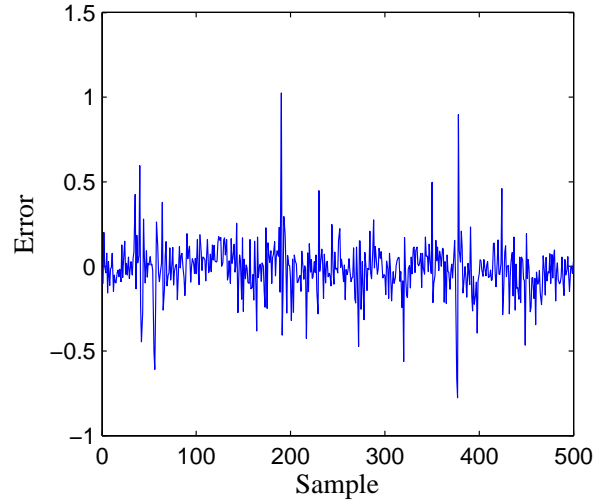

| (a) Kappa number | (b) Error |

Figure 5: Hardwood/Kappa number = 90; **Nonparametric LPV** identification (**closed-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.

(a) Kappa number   (b) Error

Figure 6: Hardwood/Kappa number = 90; **LTI** identification (**open-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.
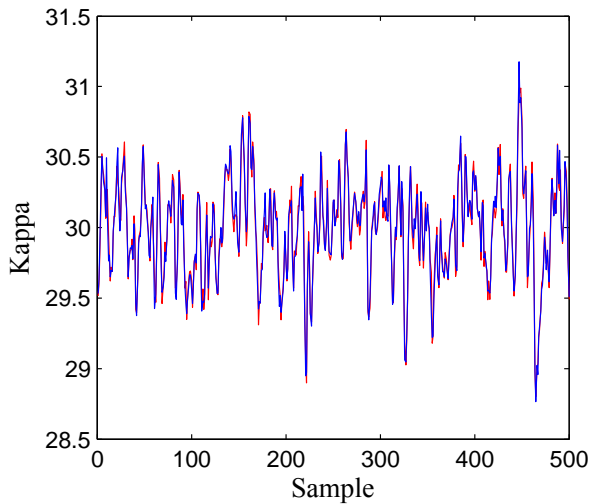

(a) Kappa number   (b) Error

Figure 7: Hardwood/Kappa number = 90; **LTI** identification (**closed-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.
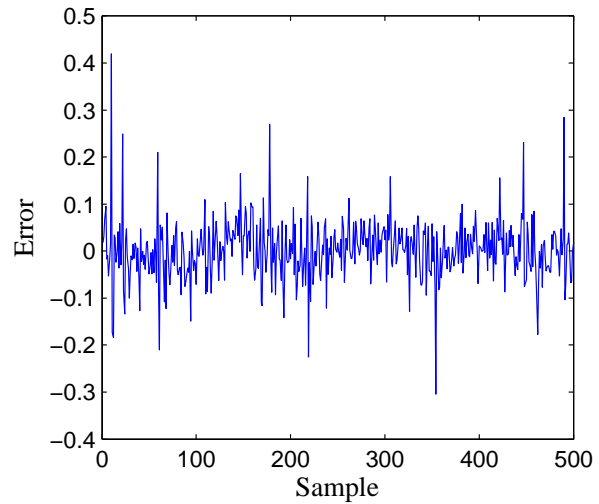
(a) Kappa number   (b) Error

Figure 8: Softwood/Kappa number = 90; **Parametric** LPV identification; (**open-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.
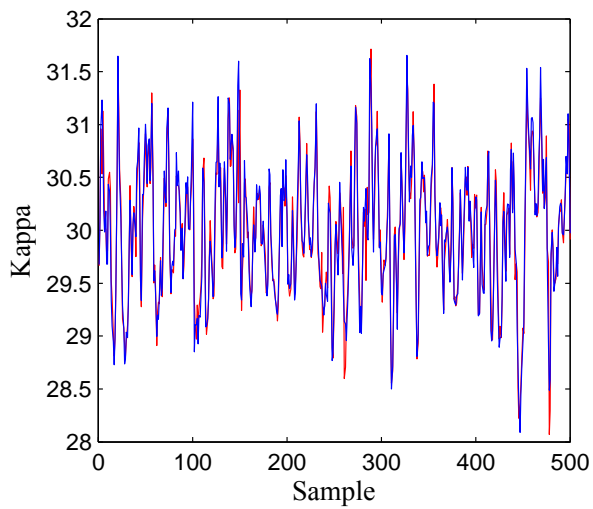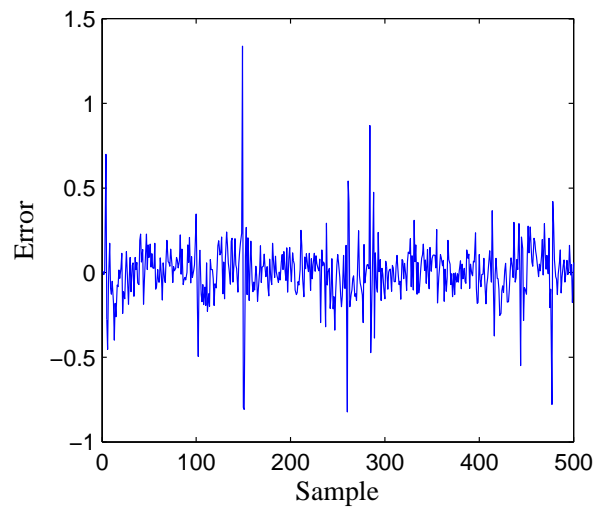

(a) Kappa number   (b) Error

Figure 9: Hardood/Kappa number = 30; **Parametric** LPV identification; (**open-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.

(a) Kappa number   (b) Error

Figure 10: Softwood/Kappa number = 30; **Parametric** LPV identification; (**open-loop**): (a) true Kappa number (blue), estimated Kappa number (red); (b) difference between true Kappa number and estimated Kappa number.