OPTIMAL POLE SELECTION FOR LPV SYSTEM IDENTIFICATION WITH OBFs, A CLUSTERING APPROACH

R. Tóth [1,2], **P.S.C. Heuberger** [1], **and P.M.J. Van den Hof**

Delft Center for Systems and Control (DCSC)
Delft University of Technology
Mekelweg 2, 2628 CD Delft, The Netherlands

Abstract: A fuzzy clustering approach is studied for optimal pole selection of Orthonormal Basis Functions (OBFs) used for the identification of Linear Parameter Varying (LPV) systems. The identification approach is based on interpolation of locally identified Linear Time Invariant (LTI) models, using globally fixed OBFs. The selection of the optimal OBF structure, that guarantees the least worst-case local modelling error, is accomplished through the joint application of the Kolmogorov n-width theory and Fuzzy c-Means (FCM) clustering of observed sample system poles. For the problem at hand, FCM solutions are given, based on three different metrics, and the qualities of the results are compared in terms of the derived OBFs.

Keywords: All pass filters; Fuzzy; Identification; Poles

1. INTRODUCTION

Many industrial applications and control systems involve phenomena that are not only functions of time, but also of other independent variables, like space coordinates. Furthermore, these effects are sometimes nonlinear and/or non-stationary. The accurate modelling of such systems is in general a complex and tedious task, involving the use of nonlinear partial differential equations, leading to models with a huge number of parameters and high computational complexity. On the other hand, accurate and efficient control of the relevant process variables is of paramount importance to satisfy the increasing performance demands.

For processes with mild non-linearities, the theory of LPV systems, generally described in a *state-space representation* (SSR) where the state-space matrices are affine functions of a time-varying, measurable, parameter vector $\zeta : \mathbb{Z} \to \Gamma$, gives a simple way to model and deal with nonlinear dynamics. Here Γ denotes the parameter space. Furthermore, control design in the LPV framework can be carried out by using LTI control theories via *gain scheduling*. Therefore, the LPV approach can offer an applicable way to meet recent industrial demands. However, existing methods for identification of such systems often produce models with high complexity. Because most control design techniques require low-order models, it is a challenge to develop efficient methods for LPV system identification that achieve low model complexity. Furthermore, serious problems may arise if the McMillan degree of the system changes due to variations of ζ, especially when the approach is based on interpolation of local models. One way of overcoming these problems is to use a fixed-order model structure on the whole Γ space.

[1] Support by the Netherlands Organisation for Scientific Research (NWO) is gratefully acknowledged.
[2] Corresponding author, email: r.t.toth@dcsc.tudelft.nl

Then, identification of the LPV system is performed by estimating multiple 'local' LTI systems around well-placed parameter set-points $\{\zeta_i\}$ of Γ (Murray-Smith and Johansen, 1997; Bamieh and Giarré, 1999; Verdult and Verhaegen, 2002). These local models are subsequently interpolated to synthesize the desired low-order LPV model. However, if the LTI model structure, used for local identification, is not linear in the parameters, the interpolation of the estimated models represents a NP-hard nonlinear optimization problem with pitfalls of local minima or existence of solution. Therefore, the choice of an easily interpolatable model structure is a turning point for the success of this identification approach.

The OBFs-based model representation offers such a structure with a well worked-out theory in the context of LTI system approximation and identification (Heuberger *et al.*, 2005). The basis functions are generated by a cascaded network of stable all-pass filters, whose pole locations represent the prior knowledge about the system at hand. This approach gives the possibility to characterize the transfer function of a strictly proper LTI system as

$$F(z) = \sum_{k=1}^{\infty} f_k g_k(z), \qquad (1)$$

where $\{f_k\}_{k=1}^{\infty}$ is the set of coefficients and $\{g_k\}_{k=1}^{\infty}$ represents the sequence of OBFs. In practice only a finite number of terms, $n \in \mathbb{N}$, is used in (1), like in *Finite Impulse Response* (FIR) models. In contrast with FIR structures, the OBF parametrization can achieve almost zero modelling error with a relatively small number of parameters, due to the infinite impulse response characteristics of the basis. Furthermore, interpolation of these models can easily take place through the interpolation of the $\{f_k\}_{k=1}^{n}$ coefficients if the set of OBFs $\{g_k(z)\}_{k=1}^{n}$ is the same for each local representation. Here, an essential challenge is to derive an OBF set, 'sufficiently rich' to describe the varying LPV dynamics at each local parameter point ζ_i, with a user-defined number of n parameters.

In practice, if the physical system is stable, it is a reasonable assumption that some sampled pole locations of the ζ-dependent pole movements of the LPV system, possibly with uncertainty bounds, are available as prior information due to local measurements. If regions Ω in the unit disc $\mathbb{D}$ are also given, where the pole variations are guaranteed to take place, then the problem of efficient OBF set selection with a pre-defined number n of basis functions, can be tackled through the usage of the Kolmogorov n-width theory for OBFs, derived by Oliveira e Silva (1996). This approach provides the selection of a set of OBFs, that ensures the least possible worst-case local modelling error for the LPV system at any point of Γ. However, determining the Ω regions from the sampled pole locations in a robust sense is not a trivial task. In this paper we aim to give a well-applicable solution, based on a FCM data clustering approach, which is capable to determine such Ω regions for which the (Oliveira e Silva, 1996) theoretical result can be applied. This solution is constructed to be the first step of a proposed approach for identifying LPV systems. This approach consists of the following steps:

(1) Determination of pole regions by FCM clustering of sampled pole locations
(2) Determination of OBFs, based on Kolmogorov n-width optimization
(3) Estimation of local models, with the optimal OBFs
(4) Interpolation of the local model coefficients

In the sequel, we present our pole clustering approach only in the context of Step 1 and 2. The paper is organized as follows: Section 2 introduces the basic mathematical description and properties of OBFs; Section 3 describes the n-width result on OBFs that we will use later on; in Section 4 we discuss the mechanism of FCM pole clustering with different approaches; in Section 5 we show through an example the applicability of the introduced methods; and in Section 6 we discuss the main results of the paper.

2. ORTHONORMAL BASIS FUNCTIONS

Because of space limitations we only consider the case of real rational SISO transfer functions. For details see (Heuberger *et al.*, 1995; Ninness and Gustafsson, 1997; Heuberger *et al.*, 2005). Let $G_0 = 1$ and $\{G_i\}_{i=1}^{\infty}$ be a sequence of inner functions (i.e. stable transfer functions with $G_i(z)G_i(\frac{1}{z}) = 1$), and let $\{A_i, B_i, C_i, D_i\}$ be balanced SSRs of G_i. Let $\{\xi_1, \xi_2, \ldots\}$ denote the collection of all poles of the inner functions. Under the (completeness) condition that $\sum_{j=1}^{\infty}(1 - |\xi_j|) = \infty$, the scalar elements of the sequence of vector functions

$$V_k(z) = (zI - A_k)^{-1} B_k \prod_{j=0}^{k-1} G_j(z), \qquad (2)$$

constitute a basis for $\mathcal{H}_{2-}(\mathbb{E})$, the *Hardy space* of functions, which are 0 for $z = \infty$, analytic on $\mathbb{E}$, the exterior of $\mathbb{D}$, and square integrable on the unit circle $\mathbb{T}$ with norm $\|.\|_{\mathcal{H}_2}$. These basis functions (2) are often referred to as the *Takenaka-Malmquist functions*. A special case of these functions is when all G_i are equal, i.e. $G_i(z) = G_b(z)\ \forall i > 0$, where G_b has McMillan degree $n_b > 0$. These cases are known as *Hambo functions* or *generalized orthonormal basis functions* (GOBFs) for arbitrary n_b, *2-parameter*

Kautz functions for $n_b = 2$, and as *Laguerre functions* for $n_b = 1$. Note that for these cases the completeness condition is always fulfilled. In the remainder of this paper we will only consider the set of Hambo functions. Let G_b be an inner function with McMillan degree $n_b > 0$ and balanced SSR $\{A_b, B_b, C_b, D_b\}$. Define $V_1(z) = (zI - A_b)^{-1}B_b$ and $\phi_j = (V_1)_j,\ j = 1, \cdots, n_b$. The Hambo basis then consists of the functions $\{\phi_j(z)G_b^i\}_{j=1,\cdots,n_b}^{i=0,\cdots,\infty}$. An important aspect of this basis is that the inner function G_b is, modulo the sign, completely determined by its poles $\{\xi_1, \cdots, \xi_{n_b}\} = \Xi_{n_b}$:

$$G_b(z) = \pm \prod_{j=1}^{n_b} \frac{1 - z\xi_j^*}{z - \xi_j}, \tag{3}$$

and it is immediate that the function V_1 has the same poles. Any $F \in \mathcal{H}_{2-}(\mathbb{E})$ can be written as

$$F(z) = \sum_{i=0}^{\infty} \sum_{j=1}^{n_b} f_{ij}\phi_j(z)G_b^i(z), \tag{4}$$

and it can be shown that the rate of convergence of this series is bounded by $\max_k |G_b(\lambda_k^{-1})|$, where $\{\lambda_k\}$ are the poles of F. In the best case, where the poles of F are the same as the poles of G_b, only the terms with $i = 0$ in (4) are non-zero.

3. KOLMOGOROV N-WIDTH FOR OBFS

Finding appropriate model sets to perform system identification is a much-studied problem with the main conclusion that in general each particular identification problem requires a model set that is tailored to the characteristics of the system to be identified. An arbitrary model set is adequate only to approximate a certain subset of $\mathcal{H}_{2-}(\mathbb{E})$, in the sense, that the model set is sufficiently rich to describe, with a relatively small number of statistically meaningful parameters, only the systems belonging to that subset. One approach to find appropriate model sets is based on the n-width concept (Pinkus, 1985), which was shown to result in appropriate model sets for robust modelling of linear systems (Mäkilä and Partington, 1993). Oliveira e Silva (1996) showed that GOBF model structures are optimal for specific subsets of systems. See also Chap. 11 in (Heuberger *et al.*, 2005). In the following, we explain the basic ingredients of this approach for discrete-time, stable, SISO systems.

Let $\mathcal{S} \subset \mathcal{H}_{2-}(\mathbb{E})$ denote the set of systems whose optimal approximation is needed. Let $\Phi = \{\phi_i\}_{i=1}^n$be a sequence of n linearly independent elements of $\mathcal{H}_{2-}(\mathbb{E})$, and let $\Psi_n = \text{Span}(\Phi)$. The distance between $F \in \mathcal{H}_{2-}(\mathbb{E})$ and Ψ_n is defined as

$$d_{\mathcal{H}_{2-}}(F, \Psi_n) = \inf_{H \in \Psi_n} \|F - H\|_{\mathcal{H}_2}. \tag{5}$$

If M_n is the collection of all n-dimensional subspaces of $\mathcal{H}_{2-}(\mathbb{E})$, then the Kolmogorov n-width of $\mathcal{S}$ in $\mathcal{H}_{2-}(\mathbb{E})$ is the smallest possible approximation error for the worst-case $F \in \mathcal{S}$,

$$w_n(\mathcal{S}, \mathcal{H}_{2-}(\mathbb{E})) = \inf_{\Psi_n \in M_n} \sup_{F \in \mathcal{S}} d_{\mathcal{H}_{2-}}(F, \Psi_n). \tag{6}$$

The subspace $\breve{\Psi}_n \in M_n$ for which w_n is minimal is called the optimal subspace in the n-width sense. A well-known result in this context is that the span of the pulse functions $\{z^{-i}\}_{i=1}^n$ is optimal for the class of stable systems of which it is only known that they are analytical in the region $(|z| > R)$, $R \in \mathbb{R}_0^+$. The worst-case approximation error is proportional to R^n.

Let G_b be an inner function with McMillan degree $n_b > 0$, and let $\{\phi_j\}_{j=1}^{n_b}$ be the first n_b Hambo functions as defined in the previous section. Denote by $\mathcal{S} \subset \mathcal{H}_{2-}(\mathbb{E})$ the set of functions that are analytic in the region $\{z, \left|G_b\left(z^{-1}\right)\right| \leq \rho\}$, and are square integrable on the boundary of that region, where $\rho > 0$ is often referred to as the *decay rate*.

Proposition 1. (Oliveira e Silva, 1996). For any $n_e \in \mathbb{N}$, Span $\left\{\phi_j(z) G_b^i(z)\right\}_{j=1,\ldots,n_b}^{i=0,\ldots,n_e-1}$ is optimal in the Kolmogorov $n = n_e \times n_b$-width sense for the set $\mathcal{S}$. The worst-case approximation error is proportional to ρ^{n_e}.

This remarkable result shows that for the specified region one can not improve on the worst-case error by adding new poles to the n_b basis poles.

In practical situations we encounter the opposite problem, referred as the *inverse Kolmogorov problem*, where a region of analycity $\Omega \subset \mathbb{D}$ is given, and we want to find the inner function G_b to describe/approximate this region in the form $\Omega(\rho, \Xi_{n_b}) = \{z, \left|G_b\left(z^{-1}\right)\right| \leq \rho\}$ with ρ as small as possible. For a given number of poles n_b, this comes down to the following min-max problem:

$$\min_{\xi_1, \cdots, \xi_{n_b}} \max_{z \in \Omega} \prod_{j=1}^{n_b} \left| \frac{z - \xi_j}{1 - z\xi_j^*} \right|. \tag{7}$$

See Chap. 10-11 in (Heuberger *et al.*, 2005) for details on this problem and solution methods. In the next section, we elaborate on the determination of the regions Ω on the basis of sampled pole locations, possibly with uncertainty bounds.

4. FUZZY POLE CLUSTERING

Objective-function-based, fuzzy clustering algorithms, such as FCM, have been extensively used in a wide collection of applications (Kaymak and Setnes, 2002). Generally, an FCM partitions the data set into overlapping groups, that describe an underlying structure within the data (Jain and Dubes, 1988). In this way, it offers the possibility

to separate the observed poles not only by hard borders, but with membership-based, overlapping areas, which incorporate not only the local, but the global data coherency. Moreover, FCM clustering does not rely on assumptions common to conventional statistical methods, such as the underlying statistical distribution of the data, and therefore it is useful in the situation of pole clustering where little prior knowledge exists.

Let $\mathbf{Z} = [z_k]_{k=1}^N \in \mathbb{D}^N$, be the set of observed poles for clustering. A cluster i is represented by its center (or *prototype*) $v_i \in \mathbb{D}$. We denote $\mathbf{V} = [v_i]_{i=1}^c$, where $c \in \mathbb{N}$ is the number of clusters. The membership matrix is $\mathbf{U} = [\mu_{ik}]_{c\times N}$, where μ_{ik} is the degree of membership of z_k to cluster i. The fuzzy constraints on $\mu_{ik} \in [0,1]$ are

$$\sum_{i=1}^{c}\mu_{ik} = 1 \text{ for } \forall k; \quad 0 < \sum_{k=1}^{N}\mu_{ik} < N \text{ for } \forall i. \quad (8)$$

For the measure of similarity, the distances d_{ik} between v_i and z_k, are computed through the so called associated metric g_i of the i^{th} cluster: $d_{ik}^2 = g_i^2(z_k, v_i)$. Generally, fuzzy pole clustering can be viewed as the minimization of the FCM-*functional* formulated as

$$J(\mathbf{V},\mathbf{U}) = \sum_{i=1}^{c}\sum_{k=1}^{N}\mu_{ik}^m d_{ik}^2, \quad (9)$$

where $m \in (1,\infty)$ is a design parameter which determines the fuzziness of the resulting partition. The minimization of (9) subject to (8) is commonly achieved by alternating optimization. The algorithm can be given as

The basic FCM algorithm

FC0 Set an initial value for $\mathbf{V}$,
FC1 Solve $\hat{\mathbf{U}} = \underset{\mathbf{U}}{\text{argmin}} J(\mathbf{V},\mathbf{U})$,
FC2 Solve $\hat{\mathbf{V}} = \underset{\mathbf{V}}{\text{argmin}} J\left(\mathbf{V},\hat{\mathbf{U}}\right)$,
FC3 If $J\left(\hat{\mathbf{V}},\hat{\mathbf{U}}\right)$ has converged, then stop.
Else set $\mathbf{V} = \hat{\mathbf{V}}$ and goto **FC1**.

This minimization approach does not guarantee that the global optimum of (9) will be reached, however convergence to –initial condition dependent– local minima always holds through the FC1 and FC2 provided descent. Furthermore, if $m \to 1$ the clustering becomes hard, each z_k only belongs to 1 cluster, however if $m \to \infty$, then each z_k becomes an equal member of every cluster. In practice, usually the value $m = 2$ is applied. For more details on the properties and mechanism of FCM clustering see Chap. 3 in (Bezdek, 1981).

By assigning different metrics for the FCM algorithm, different fuzzy partitions of $\mathbf{Z}$ can be achieved. In the literature several metrics are used for clustering purposes. We present three which we found the most applicable for pole clustering.

1. *Euclidian*, $g_{(e)}^2(x,y) = (x-y)^T(x-y)$. In clustering theory it is known that for data with two features, like $\mathrm{Re}(z_k)$ and $\mathrm{Im}(z_k)$ for the pole locations $\{z_k\}$, $g_{(e)}$ is the most effective choice among other *Minkowski* metrics. Furthermore, $g_{(e)}$ is symmetric to any feature directions, therefore the determined clusters are spherical.

2. *Mahalanobis*, $g_{(m)}^2(x,y) = (x-y)^T\Upsilon(x-y)$, where Υ is the inverse covariance matrix of the data set belonging to the cluster. Through Υ, $g_{(m)}$ adopts the shape of the cluster to the variance of the determined group of poles. This provides ellipsoidal cluster shapes.

3. *Kolmogorov*, $g_{(k)}^2(x,y) = \left|\frac{x-y}{1-xy^*}\right|^2$, a metric on $\mathbb{D}$ only. By applying $g_{(k)}$, the pole clustering and the minimization of (7) can be done in one step, in the sense, that we assign exactly one basis pole $\xi_i = v_i$, to each obtained cluster. This directly follows from the substitution of $g_{(k)}$ into (9). However, because of the symmetrical properties of $g_{(k)}$, which induces spherical clusters, and because of the assumption of 1 assigned pole per region, this approach constitutes only the simplest case of the joint mechanism of Steps 1 and 2 in the identification sequence. Here, worst-case 1-width optimality holds separately in each region and not in an n-width sense for the whole $\mathbf{Z}$. On the other hand, $g_{(k)}$-based FCM is computationally attractive and provides a fast and reliable solution for the joint clustering and Kolmogorov problem.

The determination of the number of 'natural' groups in $\mathbf{Z}$ is also important for the successful application of the previously given FCM method. Similarity-based cluster merging is frequently used for this purpose (Kaymak and Setnes, 2002). In the sequel, similarity-based merging is going to be used in our FCM solution.

By projecting the observed poles from $\mathbb{D}$ into a quasi-continuous domain of $\mathbb{C}^-$, based on the idea of discrete/continuous pole conversion: $z_k = e^{\tau_0\tilde{z}_k}$, where $\tau_0 = \min_k|z_k|$, more effective pole clustering can be achieved in the new domain. This nonlinear *quasi-continuous projection* (QCP) projects the derived linear cluster shapes from $\mathbb{C}^-$ into curves of $\mathbb{D}$, resulting in a more representative description of banana-shaped regions. QCP also preserves scatter of $\mathbf{Z}$ and magnifies distances if poles are approaching $\mathbb{T}$. QCP provides an alternative way of pole clustering, but can not be applied for $g_{(k)}$ as this is not a metric in $\mathbb{C}^-$. Furthermore, uncertainty of the observed poles can also be included in the above presented FCM algorithm by letting each z_k have a volume. This modification changes the calculation of d_{ik}^2, using average or worst-case values of the associated g_i on the volume data set.

Table 1. Results of FCM pole clustering

Test case	Euc.	Mah.	Kol.
Avrg. num. of iterations[3]: N_{av}	27	30	32
Num. of obtained clusters: c	5	5	5
Xie-Beni index[4]: χ (dB)	-34.07	-62.64	-21.64
Worst-case Kol. dis.: ρ (dB)	-11.65	-13.48	-12.58

5. RESULTS OF APPLICATION

As an example, we consider a discrete-time, asymptotically stable LPV system, which is sensitive to parameter variations. This system is given in a parameter-affine controller canonical form:

$$x(k+1) = (\mathbf{A}_0 + \mathbf{A}_1\zeta(k))\,x(k) + \mathbf{B}u(k) \quad (10)$$

$$y(k) = \mathbf{C}x(k) + \mathbf{D}u(k) \quad (11)$$

where $\zeta : \mathbb{Z} \to [-1, 1]$, is the scheduling parameter vector and the matrices are defined as

$$\mathbf{A}_0 = \begin{bmatrix} \mathbf{0}_{1\times4} & \mathbf{A}_{021} \\ \mathbf{I}_{4\times4} & \mathbf{A}_{022} \end{bmatrix}; \mathbf{A}_1 = \frac{21}{800} \cdot \begin{bmatrix} \mathbf{0}_{1\times4} & \mathbf{A}_{121} \\ \mathbf{I}_{4\times4} & \mathbf{A}_{122} \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} 1 & \mathbf{0}_{1\times4} \end{bmatrix}^T; \mathbf{C} = \begin{bmatrix} \mathbf{I}_{1\times5} \end{bmatrix}; \mathbf{D} = 0,$$

where $\mathbf{A}_{021} = 0.155$, $\mathbf{A}_{121} = 0.1$, $\mathbf{A}_{022} = \begin{bmatrix} -1.155, & 3.244, & -4.561, & 3.302 \end{bmatrix}^T$, and $\mathbf{A}_{122} = \begin{bmatrix} -0.1, & 1, & -1, & 0.1 \end{bmatrix}^T$. By fixing $\zeta(k)$ to set-point values $\{\zeta_i\}_{i=1}^{n_p} = \{-1; -1+h; \ldots; 1\}$, where $h = 0.4$, $n_p = 6$ local LTI representations of the LPV system can be obtained, whose pole locations are the samples of the ζ-dependent pole movements. These LTI systems represent in our identification approach the results of local identification.

For the obtained $N = 6 \cdot 5$ pole locations, the previously introduced metrics-based FCM algorithm was applied (with similarity-based merging) starting from $N/2$ random initial clusters and with $m = 2$. The cluster centers were let to extend in volumes; this modification is explained in (Kaymak and Setnes, 2002). Furthermore, QCP was used in the *Euclidian* (E) and the *Mahalanobis* (M) cases. The results of the algorithms are presented in Table 1 and Figure 1.

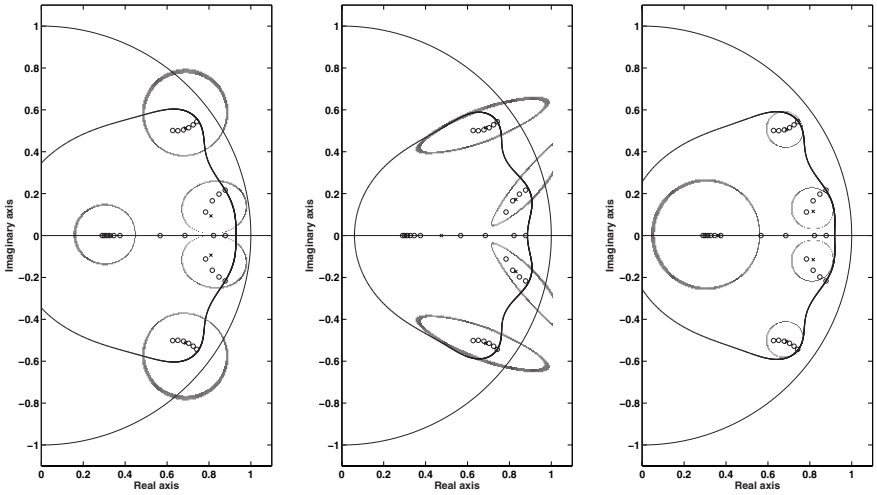
Here the cluster borders are given with bold lines. By using the cluster centers as basis poles, $\Xi_{n_b} = \mathbf{V}$, $n_b = c$, the resulting Kolmogorov region $\Omega(\rho, \Xi_c)$ is also given in the figures (thin line) with the corresponding ρ in Table 1. Based on these results, the following observations can be made:

- The differences in N_{av}[3] are relatively small, and in a few iteration steps the optimization in each case hits the minimum. However, in the (M)-case the FCM can easily get stuck in a local minimum, therefore several runs are necessary to obtain the global minimum.
- All algorithms converge to $c = 5$, which corresponds to the number of clusters by visual grouping.
- The shapes of the resulting cluster regions, computed from the membership functions by gridding, are different in each case:
  - (E) *Euclidian* case. Cluster shapes are spherical, containing unstable poles. The regions are also large near $\mathbb{T}$ introducing unnecessary uncertainty for the grouped poles. Some pole locations on the real axis are left out from the pole regions. The algorithm treats these unlikely dynamics as outliers which results in that they are only partly belonging to a cluster. This property is important in a noisy case, where outliers due to noise can compromise the effectiveness of the obtained regions.
  - (M) *Mahalanobis* case. The resulting regions are curved ellipsoidals, corresponding to a better description of the underlying pole structure. However, they also contain parts of $\mathbb{E}$. One of the clusters characterizes the real axis, directly representing real pole movements. As it can be seen, QCP enhances the resulting shapes, curving the ellipsoids due to the projection.
  - (K) *Kolmogorov case*. The cluster shapes are spherical and well placed strictly inside of $\mathbb{D}$, with very tight bounds near $\mathbb{T}$.
- χ [4] is small in all of the cases, showing that each partition represents well the underlaying structure. It is the smallest in the (M)-case which is a direct consequence of the adaptive-norm-based clustering, and it is the largest in the (K)-case due to the magnified region near the origin.
- The resulting Kolmogorov boundary with $n_b = c = 5$ OBFs is relatively tight in all cases containing the obtained clusters. ρ is also acceptable which means small modelling error if these OBFs are used for identification. In the (M)-case, ρ is the best, which is again a consequence of the adaptive cluster shapes. The (K)-case delivers the second best result due to its 1-width optimality based effectiveness making it superior among other symmetrical-metrics-based FCM solutions.

It is important to note that by applying the n-width result on the obtained Ω regions, represented by the cluster boundaries restricted to $\mathbb{D}$, an even better selection of OBFs can be achieved, if more poles are assigned to clusters

[3] N_{av} is based on the results of 10 runs starting from random initial clusters.

[4] For checking the validity of the partition the Xie-Beni validity index χ (Xie and Beni, 1991) was applied, which gives a common ground of comparison between different metrics driven FCMs. The smaller the value of χ, the better the corresponding fit.



(a) Euclidian-FCM (b) Mahalanobis-FCM (c) Kolmogorov-FCM

Fig. 1. Results of FCM clustering: sampled poles (o) and resulting cluster centers (×), cluster boundaries (bold lines), and Kolmogorov boundaries (thin lines).

with large volumes or complicated shapes. This is accomplished through the mechanism described in (Heuberger *et al.*, 2005) in Chap. 11, where through a nonlinear optimization with high computational load the desired n-width optimal OBFs can be determined.

6. CONCLUSION

The FCM-based pole clustering, presented in this paper, offers an attractive procedure for determining pole regions based on local observations of an LPV system. The determined regions can be used to synthesize an effective set of OBFs in the Kolmogorov n-width sense for fixed-order local identification of the physical process. This contribution enables the use of the n-width result to determine an optimal set of OBFs for this specific LPV identification approach.

However, to derive a method, which is capable of the joint application of the n-width result and the pole clustering still remains an open question. It is believed that such a method can be derived from incorporating the n-width theory directly into the FCM mechanism, like the 1-width optimality of the (K)-case. The results presented in this paper are a first step towards this joint solution.

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