Linear Parameter-Varying Control of a Copolymerization Reactor

Hossam S. Abbas * **Sandy Rahme** ** **Nader Meskin** **
Christian Hoffmann *** **Roland Tóth** ****
Javad Mohammadpour †

Electrical Engineering Dept., Faculty of Engineering, Assiut University, 71515 Assiut, Egypt, (e-mail: hossam.abbas@aun.edu.eg).
** *Dept. of Electrical Engineering, College of Engineering, Qatar University, PO Box 2713 Doha, Qatar, (e-mail: {sandy.rahme,nader.meskin}@qu.edu.qa)*
*** *Institute of Electrical Engineering in Medicine, Universität zu Lübeck, 23558 Lübeck, Germany (christian.hoffmann@uni-luebeck.de)*
**** *Control Systems Group, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, (r.toth@tue.nl)*
† *Complex Systems Control Lab, College of Engineering, The University of Georgia, Athens, GA 30602, USA, (javadm@uga.edu)*

Abstract: This paper demonstrates the application of the *linear parameter-varying* (LPV) framework to control a copolymerization reactor. An LPV model representation is first developed for a nonlinear model of the process. The LPV model complexity in terms of the model order and the number of scheduling variables is then reduced by truncating those system states that have insignificant direct influence on the input-output behavior of the system and do not directly appear in the output equations of the model. It is important to note that these truncated states are still preserved in the reduced model by affecting the scheduling parameters and hence enabling the representation of the input-output map. Using the derived model, a *linear fractional transformation* (LFT) based LPV controller synthesis approach is used to synthesize a controller for the process. Simulation based studies of the closed-loop behavior of the system regulated by the designed LPV controller demonstrate that the LPV controller solution outperforms a model predicative control designed previously for this system in terms of the achieved control performance and the online computational effort.

1. INTRODUCTION

From an industrial perspective, there are interesting incentives for efficient control of polymer reactors. These include maximization of production quality and rates and minimization of transition losses. Control of polymerization reactors is a difficult task due to several challenges specific to them. Basically, they are highly *nonlinear* (NL) systems whose states exhibit strongly coupled behavior (Soroush and Kravari, 1993), (Özkana et al., 2003), and hence they exhibit multiple steady states, as well as oscillatory or unstable modes at some operating points (Congalidis and Richards, 1998). They can be highly exothermic and result in a reactor thermal runaway in the absence of an appropriate control strategy. Several control approaches have been investigated for these processes, see (Richards and Congalidis, 2006). Classical control methods such as standard PID and multi-loop PID control, e.g., (Congalidis et al., 1989), are commonly used as they require minimal process knowledge. However, they are not adequate to cope with such a multivariate control problem with strong interactions between the controlled variables. A further relevant control approach relies on model predictive control techniques based on simple process models, e.g., (Maner and Doyle, 1997), (Özkana et al., 2003), that allow a rapid

transition between different operating points. Nonlinear control (Soroush and Kravari, 1993) has been considered as well, which depends on the availability of a highly accurate nonlinear model and online measurements of *time-varying* (TV) model parameters.

Generally, optimal control techniques are preferred if a good process model is available because of their presumed superior performance (Embirucu et al., 1996). Moreover, adaptive control strategies can be applied in order to take the time-varying nature of the process into account, provided that online measurements/estimations are available. *Linear parameter-varying* (LPV) control (Mohammadpour and Scherer, 2012) is a promising candidate for control design that can be classified as an adaptive control technique based on the extension of powerful linear time-invariant (LTI) approaches such as $\mathcal{H}_2/\mathcal{H}_\infty$ optimal control, see e.g., (Mohammadpour and Scherer, 2012), to address the control design problem for NL and TV systems. LPV systems are represented by dynamical models capable of describing TV behaviors in terms of a linear structure depending on the so-called scheduling variables, which are assumed to be online measurable. The variation of the scheduling variables represents time-variance, changing operating conditions, etc. The capability of LPV framework to model and control NL/TV systems has been

demonstrated in terms of several successful applications, e.g., (Abbas et al., 2014) and (Bachnas et al., 2014).

In this paper, we investigate an LPV control design strategy to control a free radical solution polymerization in a *continuously stirred tank reactor* (CSTR). To implement the LPV controller, an LPV representation in state-space form of the nonlinear model developed in (Congalidis et al., 1989) is first introduced. The complexity of the model is reduced by a specific model reduction approach which also preserves the input-output behavior of the full model. Then, the operating region is specified and the control design approach introduced in (Scherer, 2001) is utilized to design an LPV controller based on the mixed sensitivity design. The objectives of this study are twofold: (1) Show a practically applicable LPV modeling technique and complexity reduction approach for copolymerization processes. (2) Assess the capabilities of LPV control to providing high performance control solution for polymerization reactors.

The paper is organized as follows: In Section 2, the NL dynamical model of the copolymerization reactor is reviewed and an LPV state-space model representation of the process is developed. In Section 3, a low-complexity LPV representation for the process is developed. The synthesis of an LPV controller and the closed-loop implementation are demonstrated in Section 4. Finally, conclusions are drawn in Section 5. Notations considered in this paper are standard. The subscripts a, b, i, s, t, z are used to indicate monomer A, monomer B, initiator, solvent, chain transfer, and inhibitor, respectively. The symmetric completion of a matrix is denoted by $*$ and $\ker[X]$ denotes the nullspace of a matrix X.

2. PRELIMINARIES

2.1 Nonlinear Dynamic Model of The Process

In this paper, we consider the model of the solution copolymerization reactor developed in (Congalidis et al., 1989); a flow diagram of the process is shown in Fig. 1. Monomers A and B, which are methyl methacrylate and vinyl acetate, respectively, are continuously added with initiator, solvent, and chain transfer agent, which are benzene, azobisisobutyronitrile and acetaldehyde, respectively. Moreover, an inhibitor, which is mdinitrobenzene, may enter with the fresh feed as an impurity. The reactor is assumed to be a jacketed, well-mixed tank. A coolant flows through the jacket to remove the heat of polymerization. The NL mathematical model is based on a free radical mechanism with 27 separate reactions, and is given by
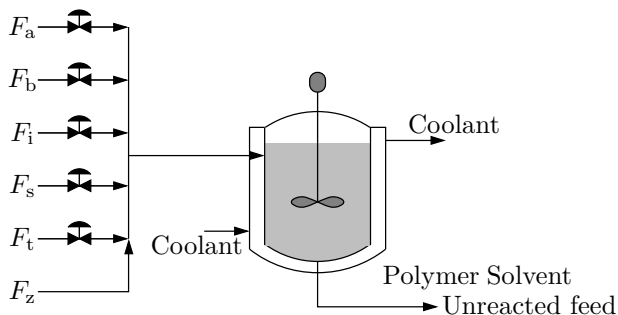

Fig. 1. Copolymerization reactor

$$\frac{\mathrm{d}}{\mathrm{d}t}C_k = -\phi_l C_k + \frac{F_k}{M_k V} - \frac{\sum_k F_k}{V\rho}C_k, \tag{1a}$$
$$(k,l) = \{(a,1),(b,2),(i,3),(s,4),(t,5),(z,6)\},$$
$$\frac{\mathrm{d}}{\mathrm{d}t}T_r = \frac{\phi_7}{c\rho}C_a + \frac{\phi_8}{c\rho}C_b - \frac{SU}{Vc\rho}(T_r-T_j) + \frac{\sum_k F_k}{V\rho}(T_{rf}-T_r), \tag{1b}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\lambda_a = \phi_1 C_a - \frac{\sum_k F_k}{V\rho}\lambda_a, \tag{1c}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\lambda_b = \phi_2 C_b - \frac{\sum_k F_k}{V\rho}\lambda_b, \tag{1d}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_0 = \phi_9 C_a + \phi_{10}C_b - \frac{\sum_k F_k}{V\rho}\psi_0, \tag{1e}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_1 = \phi_{11}C_a + \phi_{12}C_b - \frac{\sum_k F_k}{V\rho}\psi_1, \tag{1f}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_2 = \phi_{13}C_a + \phi_{14}C_b - \frac{\sum_k F_k}{V\rho}\psi_2, \tag{1g}$$

where the inputs are the flow rates $F_a, F_b, F_i, F_s, F_t, F_z$ in kg/h, and the reactor jacket temperature T_j is in K. The states are the concentrations $C_a, C_b, C_i, C_s, C_t, C_z$ in kmol/m^3, the reactor temperature T_r in K, the molar concentrations of monomer in polymer λ_a, λ_b and the moments of molecular weight distribution are ψ_0, ψ_1, ψ_2. The variables $\phi_i, i \in \{1, \cdots, 14\}$ are NL functions of the states (see Appendix A for their definition). The constant parameters are the molecular weights $M_a, M_b, M_i, M_s, M_t, M_z$, the reactor volume V, the heat surface area S, the overall heat transfer coefficient U, the heat capacity c, the density ρ and the reactor feed temperature T_{rf}; see Table 1 for the value of these constant parameters. The important reactor output variables for control are T_r and

$$G_{pi} = M_a V \phi_1 C_a + M_b V \phi_2 C_b, \tag{2a}$$
$$Y_{ap} = \phi_{15}\lambda_a, \tag{2b}$$
$$M_{pw} = \phi_{16}\psi_2, \tag{2c}$$

where G_{pi}, Y_{ap}, M_{pw} are the polymer production rate in kg/h, mole fraction of monomer A in the polymer and the weight average molecular weight in kg/kmol, respectively; for the definition of ϕ_{15} and ϕ_{16}, see Appendix A.

Table 1. Constant parameters for the model

M_a	100.10	kg/kmol	V	1	m^3
M_b	86.09	kg/kmol	S	4.6	m^2
M_i	164.00	kg/kmol	U	216.00	J/m^2·s·K
M_s	78.11	kg/kmol	c	2.01	kJ/kg·K
M_t	44.05	kg/kmol	ρ	879.00	kg/m^3
M_z	168.11	kg/kmol	T_{rf}	353.00	K

The main control objective in (Özkana et al., 2003) was to provide a fast transition between two operating points as shown in Table 2 with a controller that is robust against unmeasured disturbances. The first operating point, OP1, given in (Congalidis et al., 1989) was obtained for a monomer feed ratio $F_A/F_B = 0.2$ while the second one, OP2, was obtained by increasing the ratio by 0.25 keeping F_B constant. In order to achieve this objective, the manipulated variables to control the four previously specified output variables are chosen –based on the investigation in (Congalidis et al., 1989)– to be F_a, F_b, F_t and T_j. For comparison purposes, we consider in this work the same

control objective and associated variables; the other inputs are kept constant as shown in Table 1.

Table 2. Operating conditions for the process

	OP1	OP2
G_{pi} (kg/h)	23.35	24.9
Y_{ap}	0.56	0.64
M_{pw} (10^4kg/kmol)	3.4	3.9
T_{r} (K)	353.06	353.3

2.2 LPV State-Space Representation

The design of an LPV gain-scheduled controller for the copolymerization reactor is based on an LPV state-space representation of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}x = \mathcal{A}(\theta)x + \mathcal{B}(\theta)u, \tag{3a}$$

$$y = \mathcal{C}(\theta)x + \mathcal{D}(\theta)u, \tag{3b}$$

where $u : \mathbb{T} \to \mathbb{R}^{n_{\mathrm{u}}}$, $y : \mathbb{T} \to \mathbb{R}^{n_y}$, $x : \mathbb{T} \to \mathbb{R}^n$ are the input, output, and state of the system, respectively, and $\mathbb{T} = \mathbb{R}_{\mathrm{o}}^+$ is the time domain. The mappings $\mathcal{A}(\cdot)$, $\mathcal{B}(\cdot)$, $\mathcal{C}(\cdot)$ and $\mathcal{D}(\cdot)$ are continuous matrix-valued functions of a set of variables referred to as scheduling variables vector $\theta \in \mathbb{P}_\theta$, where $\mathbb{P}_\theta \subseteq \mathbb{R}^{n_\theta}$ denotes the scheduling regime which is assumed to be compact. θ is a function of a measurable signal vector $p : \mathbb{T} \to \mathbb{R}^{n_{\mathrm{P}}}$ in the system and this functional dependence is expressed as $\theta = g(p)$, where $g : \mathbb{R}^{n_{\mathrm{P}}} \to \mathbb{R}^{n_\theta}$ is a continuous mapping. The scheduling set $\mathbb{P}_\theta$ is assumed to be overapproximated by a polytope, defined as the convex hull given by the vertices θ_{v_i}, i.e.,

$$\mathbb{P}_\theta := \mathrm{Co}\{\theta_{\mathrm{v}_1}, \theta_{\mathrm{v}_2}, \dots, \theta_{\mathrm{v}_{n_{\mathrm{v}}}}\}, \tag{4}$$

where $n_{\mathrm{v}} = 2^{n_\theta}$, and $\mathbb{P}_\theta$ is based on the convex hull of the bounds of θ_i. An LPV state-space representation is called to have affine scheduling dependence if the model depends affinely on θ. For the representation (3), this means that any of the matrices $\mathcal{A}, \dots, \mathcal{D}$ can be represented as

$$M^{\mathrm{a}}(\theta) = M_0^{\mathrm{a}} + \theta_1 M_1^{\mathrm{a}} + \dots + \theta_{n_\theta} M_{n_\theta}^{\mathrm{a}}. \tag{5}$$

Since θ can be expressed as a convex combination of n_{v} vertices θ_{v_i}, if (5) holds true, it follows that the system can be represented by a linear combination of LTI models at the vertices. This is called a *polytopic* LPV state-space representation, where for any of the matrices $\mathcal{A}, \dots, \mathcal{D}$ we have $M^{\mathrm{p}}(\theta) = \sum_{i=1}^{n_{\mathrm{v}}} \alpha_i M^{\mathrm{p}}(\theta_{\mathrm{v}_i})$, such that $\sum_{i=1}^{n_{\mathrm{v}}} \alpha_i = 1$ and $\alpha_i \geq 0$ are the convex coordinates.

3. LPV MODELING OF THE PROCESS

The NL model (1) depends affinely on the NL functions $\phi_1, \dots, \phi_{16}$, and hence, one can easily rewrite (1) in the form (3) by considering $\phi_1, \dots, \phi_{16}$ and $\sum_k F_k$ to be the scheduling variables that correspond to $\theta \in \mathbb{R}^{17}$. In this case, θ is a vector-valued function of the inputs and states of the system which is a function of the measurable signal vector p according to

$$p = \begin{bmatrix} F_{\mathrm{a}} & F_{\mathrm{b}} & F_{\mathrm{i}} & F_{\mathrm{s}} & F_{\mathrm{t}} & F_{\mathrm{z}} & C_{\mathrm{a}} & C_{\mathrm{b}} & C_{\mathrm{i}} & C_{\mathrm{s}} & C_{\mathrm{t}} & C_{\mathrm{z}} & T_{\mathrm{r}} & \lambda_{\mathrm{a}} & \lambda_{\mathrm{b}} & \psi_1 \end{bmatrix}^\top. \tag{6}$$

The scheduling set $\mathbb{P}_\theta$ can be defined by obtaining bounds on θ based on operational limits of p, which can be computed according to the operating region defined in Table 2. This provides an exact polytopic LPV representation for

(1). However, it could be too complex for LPV control synthesis to achieve a specific desired control performance due to the large number of scheduling variables, which is 17 here. Based on the observations reported in (Hoffmann and Werner, 2014), the large number of scheduling variables renders the synthesis problem intractable due to the large number of underlying matrix inequalities (Apkarian et al., 1995) or decision variables (Scherer, 2001). Furthermore, even if rendered tractable in an LFT framework, the necessary structural constraints may render the resulting control performance overly conservative. Finally, online controller implementation may turn out to be excessively costly. Therefore, it is necessary to obtain an LPV model with a small number of scheduling variables and a reduced order.

In the following, we reduce the complexity of the NL model (1) in terms of its dynamical order so that it can be used to obtain a low-complexity LPV model, i.e., with a reduced order and reduced number of scheduling variables. For the operating region that is considered here (see Table 2), it has been shown in (Maner and Doyle, 1997) that closing the temperature loop with a PI controller, i.e., assigning the manipulated variable T_{j} to only control T_{r}, yields the system well-conditioned, which can provide a full exploration of the interaction compensation abilities of multivariable control. Safety reasons could be a justification for closing the temperature loop to prevent the reactor runaway. By closing the temperature loop, in (Maner and Doyle, 1997), a well-conditioned 3×3 control problem has been obtained, which results in a better control performance compared to the 4×4 control problem. Therefore, (1b) can be truncated, which reduces the dynamical order of the model by one and removes the functions ϕ_8 and ϕ_9 without any approximation as the reduced model still can receive information of T_{r} via the other functions, see Appendix A.

Next, we further reduce the dynamical order of the obtained 3×3 representation of the model (1), i.e., the model (1) without (1b) and with the outputs (2), such that the exact input-output behavior between the full and reduced models is preserved. The idea is to truncate all the states that do not explicitly affect the outputs of the model. By observing (2), it can be seen that the three outputs of the model are directly affected by the states $C_{\mathrm{a}}, C_{\mathrm{b}}, \lambda_{\mathrm{a}}, \psi_2$. Therefore, the remaining states of the original model can be truncated, and hence, we can obtain the following reduced model

$$\frac{\mathrm{d}}{\mathrm{d}t}C_{\mathrm{a}} = -\phi_1 C_{\mathrm{a}} + \frac{F_{\mathrm{a}}}{M_{\mathrm{a}}V} - \frac{F_{\mathrm{isz}}}{V\rho}C_{\mathrm{a}} - \frac{(F_{\mathrm{a}}+F_{\mathrm{b}}+F_{\mathrm{t}})}{V\rho}C_{\mathrm{a}}, \tag{7a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}C_{\mathrm{b}} = -\phi_2 C_{\mathrm{b}} + \frac{F_{\mathrm{b}}}{M_{\mathrm{b}}V} - \frac{F_{\mathrm{isz}}}{V\rho}C_{\mathrm{b}} - \frac{(F_{\mathrm{a}}+F_{\mathrm{b}}+F_{\mathrm{t}})}{V\rho}C_{\mathrm{b}}, \tag{7b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\lambda_{\mathrm{a}} = \phi_1 C_{\mathrm{a}} - \frac{F_{\mathrm{isz}}}{V\rho}\lambda_{\mathrm{a}} - \frac{(F_{\mathrm{a}}+F_{\mathrm{b}}+F_{\mathrm{t}})}{V\rho}\lambda_{\mathrm{a}}, \tag{7c}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_2 = \tilde{\phi}_{13} C_{\mathrm{a}} - \frac{F_{\mathrm{isz}}}{V\rho}\psi_2 - \frac{(F_{\mathrm{a}}+F_{\mathrm{b}}+F_{\mathrm{t}})}{V\rho}\psi_2, \tag{7d}$$

with the outputs in (2), where

$$\tilde{\phi}_{13} = \frac{\phi_{13}C_{\mathrm{a}} + \phi_{14}C_{\mathrm{b}}}{C_{\mathrm{a}}}, \tag{8}$$

and $F_{\text{isz}} = F_i + F_s + F_z$, which is assumed to be a constant parameter as it is composed of non-manipulated variables. The variations of F_i, F_s and F_z during operation are considered to be unmeasured disturbances. In (8), $C_a = 0$ is prohibitive as it is meaningless to have zero concentration of polymer A, and hence, (8) remains bounded in the meaningful operational regime.

Now, an exact LPV representation for (7) with (2) can be written as in (3) where the state matrices in terms of ϕ_i are given by

$$
\mathcal{A}(\cdot) = \begin{bmatrix}
-(\phi_1 + \dfrac{F_{\text{isz}}}{V\rho}) & 0 & 0 & 0 \\
0 & -(\phi_2 + \dfrac{F_{\text{isz}}}{V\rho}) & 0 & 0 \\
\phi_1 & 0 & -\dfrac{\sum_k F_k}{V\rho} & 0 \\
\tilde{\phi}_{13} & 0 & 0 & -\dfrac{F_{\text{isz}}}{V\rho}
\end{bmatrix},
$$
(9a)

$$
\mathcal{B}(\cdot) = \begin{bmatrix}
\dfrac{1}{M_a V} - \dfrac{C_a}{V\rho} & -\dfrac{C_a}{V\rho} & -\dfrac{C_a}{V\rho} \\
-\dfrac{C_b}{V\rho} & \dfrac{1}{M_b V} - \dfrac{C_b}{V\rho} & \dfrac{C_b}{V\rho} \\
0 & 0 & 0 \\
-\dfrac{\psi_2}{V\rho} & -\dfrac{\psi_2}{V\rho} & -\dfrac{\psi_2}{V\rho}
\end{bmatrix},
$$
(9b)

$$
\mathcal{C}(\cdot) = \begin{bmatrix}
M_a V \phi_1 & M_b V \phi_2 & 0 & 0 \\
0 & 0 & \phi_{15} & 0 \\
0 & 0 & 0 & \phi_{16}
\end{bmatrix}, \qquad \mathcal{D} = 0,
$$
(9c)

where $\sum_k F_k = F_{\text{isz}} + F_A + F_B + F_C$. Note that for the representation (9), the functions $\phi_1, \phi_2, \tilde{\phi}_{13}, \phi_{15}, \phi_{16}$ and the states C_a, C_b, ψ_2 constitute the scheduling vector θ resulting in total 8 scheduling variables. Next, we further reduce this dimension by introducing a new input vector

$$
\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \underbrace{\begin{bmatrix}
\dfrac{1}{M_a V} - \dfrac{C_a}{V\rho} & -\dfrac{C_a}{V\rho} & -\dfrac{C_a}{V\rho} \\
-\dfrac{C_b}{V\rho} & \dfrac{1}{M_b V} - \dfrac{C_b}{V\rho} & \dfrac{C_b}{V\rho} \\
-\dfrac{\psi_2}{V\rho} & -\dfrac{\psi_2}{V\rho} & \dfrac{\psi_2}{V\rho}
\end{bmatrix}}_{\mathcal{E}} \begin{bmatrix} F_a \\ F_b \\ F_t \end{bmatrix}.
$$
(10)

The transformation matrix $\mathcal{E}$ is non-singular for all values of C_a, C_b and ψ_2 is in the specified operating range. Then, a new $\mathcal{B}$ matrix for the LPV model can be introduced as

$$
\mathcal{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$
(11)

in terms of the new input vector (10) considered for the model. Note that this yields a constant B matrix. Moreover, we define new outputs as

$$y_1 = G_{\text{pi}},$$
(12a)
$$y_2 = Y_{\text{ap}}/\phi_{15} = Y_{\text{ap}}(\lambda_a + \lambda_b),$$
(12b)
$$y_3 = M_{\text{pw}}/\phi_{16} = M_{\text{pw}}\psi_1,$$
(12c)

and hence, a new C matrix for the model can be introduced as

$$
\mathcal{C}(\cdot) = \begin{bmatrix}
M_a V \phi_1 & M_b V \phi_2 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
$$
(13)

Introducing the new inputs and outputs results in a LPV representation (3) of relatively low complexity for the NL model with state-space matrices given by

$$
\mathcal{A}(\theta) = \begin{bmatrix}
-(\theta_1 + \dfrac{F_{\text{isz}}}{V\rho}) & 0 & 0 & 0 \\
0 & -(\theta_2 + \dfrac{F_{\text{isz}}}{V\rho}) & 0 & 0 \\
\theta_1 & 0 & -(\dfrac{\theta_4}{V\rho} + \dfrac{F_{\text{isz}}}{V\rho}) & 0 \\
\theta_3 & 0 & 0 & -\dfrac{F_{\text{isz}}}{V\rho}
\end{bmatrix},
$$
(14)

(11), (13) and $\mathcal{D} = 0$, where $\theta_1 = \phi_1$, $\theta_2 = \phi_2$, $\theta_3 = \tilde{\phi}_{13}$, $\theta_4 = F_a + F_b + F_t$; $\theta_1, \cdots, \theta_4$ compose the entries of the scheduling vector θ. Taking into consideration (10) and (12), the LPV representation (3) with (14) has the same input-output map as of the NL model (1). Note that θ_i are functions of the preserved variables collected in p as described by (6).

Next, the parameter set $\mathbb{P}_\theta$ is defined based on the bounds of θ, see (4). First, we choose an initial range [1] for the manipulated inputs F_a, F_b and F_t, and we refer to such range as input range. Then, we grid the input range to produce a set of grid points; these can then be used to generate a set of operating points by computing the corresponding steady-state values of the state vector of (1), such that the operating points defined in Table 2 are covered. Finally, a set of values of θ is computed for each operating point, based on which the bounds of θ can be computed and hence $\mathbb{P}_\theta$ is defined. The input range can be redefined after the control synthesis step according to the closed-loop operation. It turns out that the input range

$$18 \le F_a \le 22.5, \quad 87 \le F_b \le 93, \quad 1 \le F_t \le 4 \quad \text{kg/h},$$

is sufficient to define $\mathbb{P}_\theta$, based on the bounds of θ that are obtained as

$$
\begin{aligned}
& 0.1618 \le \theta_1 \le 0.1771, \quad 0.4823 \le \theta_2 \le 0.5861, \\
& 0.0170 \le \theta_3 \le 0.0202, \quad 2.6124 \le \theta_4 \le 4.0512.
\end{aligned}
$$
(15)

Finally, it is important to point out that feasibility of LPV control design based on the derived LPV model, depends on the possibility to measure or estimating p in (6) at every time instant. This is assumed here in order to assess the performance of LPV control on the underlying process. It has been shown in (Richards and Congalidis, 2006), (Soroush and Kravari, 1993) that most of the elements of p can be measured or estimated in real time, hence fulfilling the taken assumptions is also practically feasible.

4. LPV CONTROL

4.1 LPV Control Synthesis

This section presents the design of an LPV controller with a fixed Lyapunov function using an $\mathcal{H}_\infty$ loop-shaping approach based on the gain-scheduling LPV synthesis results of (Scherer, 2001). The LPV-LFT gain-scheduling approach is considered here since it provides a versatile LPV controller synthesis framework capable of handling plants with rational parameter-dependency and potentially a large number of parameters while maintaining

[1] One can initially use the control input levels that have been observed in (Özkana et al., 2003) and (Congalidis et al., 1989).

low implementation complexity through affinely scheduled controllers in the case of plants with affine parameter-dependency (Hoffmann and Werner, 2014). The method requires the formulation of the LPV representation (3) in a *linear fractional transformation* (LFT) form as

$$\begin{bmatrix} \mathcal{A}(\theta) & \mathcal{B}(\theta) \\ \mathcal{C}(\theta) & \mathcal{D}(\theta) \end{bmatrix} = \begin{bmatrix} A & B_u \\ C_y & D_{yu} \end{bmatrix} + \begin{bmatrix} B_\theta \\ D_{y\theta} \end{bmatrix} \Theta(I - D_{\theta\theta}\Theta)^{-1} \begin{bmatrix} C_\theta & D_{\theta u} \end{bmatrix}$$

(16)

where $\Theta = \mathrm{diag}(\theta_1 I_{r1}, \cdots, \theta_{n_\theta} I_{rn_\theta})$ is a parameter matrix that includes all the scheduling variables. Note that, parameter-affine LPV models, $D_{\theta\theta} = 0$. Moreover, for the derived LPV model of the copolymerization reactor, we have $D_{yu} = 0$, $\Theta = \mathrm{diag}(\theta_1, \cdots, \theta_4) \in \mathbb{R}^{4\times 4}$. Since both the plant and the controller are time-varying systems, the $\mathcal{H}_\infty$ norm is interpreted in terms of the induced $\mathcal{L}_2$-gain.

The design objective considered here is to stabilize the closed-loop system in the operating range defined in Table 2 with a fast tracking capability and disturbance rejection taking into consideration the control input constraints as (Özkana et al., 2003)

$$0 \leq \frac{F_A}{F_B}, \leq \frac{36}{90}, \qquad 0 \leq \frac{F_C}{F_B} \leq \frac{5.4}{90},$$
$$0 \leq F_B \leq 180 \,\mathrm{kg/h}, \qquad 317.754 \leq T_j \leq 388.366 \,\mathrm{K}.$$

A standard mixed sensitivity loop-shaping approach is adopted here to meet the design objectives. The generalized plant is shown in Fig. 2. In order to shape the closed-loop sensitivity and control sensitivity functions, the weighting filters

$$W_S = \mathrm{diag}\left(\frac{8.326\times 10^{-2}}{s+9.02\times 10^{-4}}, \frac{1.088\times 10^{-1}}{s+1.01\times 10^{-3}}, \frac{1.193\times 10^{-1}}{s+1.09\times 10^{-5}}\right)$$

$$W_{KS} = \mathrm{diag}\left(\frac{1642s+3.885\times 10^4}{s+2.367\times 10^4}, \frac{641.7s+6.205\times 10^4}{s+9.67\times 10^4},\right.$$
$$\left.\frac{1679s+1243}{s+740.5}\right)$$

are used, respectively. The sensitivity weighting filter W_S has been tuned to satisfy the closed-loop bandwidth to provide a desired fast response and to drive the steady-state error towards zero. By freezing the scheduling variables vector θ of the LPV model at different values in the scheduling set $\mathbb{P}_\theta$, a set of LTI models has been obtained, from which the required bandwidth has been inferred. On the other hand, the control sensitivity weighting filter W_{KS} has been adjusted to impose an upper bound on the control sensitivity to restrict the control effort and impose a limit on the sensitivity function peak, resulting in a reduction in overshoot. Given W_S and W_{KS}, an LFT formulation of the generalized plant is obtained as

$$\begin{bmatrix} \mathcal{A}(\tilde\theta) & \tilde{\mathcal{B}}_p(\theta) & \tilde{\mathcal{B}}(\theta) \\ \tilde{\mathcal{C}}_p(\theta) & \tilde{\mathcal{D}}_{pp}(\theta) & \tilde{\mathcal{D}}_{pu}(\theta) \\ \mathcal{C}(\theta) & \tilde{\mathcal{D}}_{yp}(\theta) & \tilde{\mathcal{D}}(\theta) \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B}_p & \tilde{B}_u \\ \tilde{C}_p & \tilde{D}_{pp} & \tilde{D}_{pu} \\ \tilde{C}_y & \tilde{D}_{yp} & \tilde{D}_{yu} \end{bmatrix} +$$
$$\begin{bmatrix} \tilde{B}_\theta \\ \tilde{D}_{p\theta} \\ \tilde{D}_{y\theta} \end{bmatrix} \Theta \begin{bmatrix} \tilde{C}_\theta & \tilde{D}_{\theta p} & \tilde{D}_{\theta u} \end{bmatrix}. \quad (17)$$

Next, a condition for the existence of a gain-scheduled controller is reviewed.
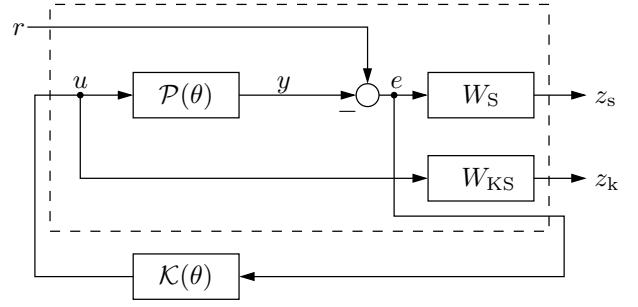

Fig. 2. Generalized plant interconnected with the LPV controller.

Theorem 1. (Scherer, 2001) There exists an LPV controller in an LFT form, such that the closed-loop system in Fig. 2 is internally stable for all $\theta \in \mathbb{P}_\theta$, if there exists positive definite symmetric matrices $X > 0$, $Y > 0$ of appropriate size and multipliers $M = M^\top$, $N = N^\top$ that satisfy the *linear matrix inequality* (LMI) conditions

$$V_X^\top \begin{bmatrix} * \\ * \\ * \\ * \end{bmatrix}^\top \begin{bmatrix} 0 & X & 0 & 0 \\ X & 0 & 0 & 0 \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & \Gamma \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ \tilde{A} & \tilde{B}_\theta & \tilde{B}_p \\ 0 & I & 0 \\ \tilde{C}_\theta & 0 & \tilde{D}_{\theta p} \\ 0 & 0 & I \\ \tilde{C}_p & \tilde{D}_{p\theta} & \tilde{D}_{pp} \end{bmatrix} V_X < 0, \quad (18a)$$

$$V_Y^\top \begin{bmatrix} * \\ * \\ * \\ * \end{bmatrix}^\top \begin{bmatrix} 0 & Y & 0 & 0 \\ Y & 0 & 0 & 0 \\ 0 & 0 & N & 0 \\ 0 & 0 & 0 & \Gamma^{-1} \end{bmatrix} \begin{bmatrix} -\tilde{A}^\top & -\tilde{C}_\theta^\top & -\tilde{C}_p^\top \\ I & 0 & 0 \\ -\tilde{B}_\theta^\top & 0 & -\tilde{D}_{p\theta}^\top \\ 0 & I & 0 \\ -\tilde{B}_p^\top & -\tilde{D}_{\theta p}^\top & -\tilde{D}_{pp}^\top \\ 0 & 0 & I \end{bmatrix} V_Y > 0,$$

(18b)

$$[*]^\top M \begin{bmatrix} I \\ \Theta \end{bmatrix} > 0, \quad [*]^\top N \begin{bmatrix} -\Theta^\top \\ I \end{bmatrix} < 0, \quad \forall \theta \in \mathbb{P}_\theta \quad (18c)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \quad (18d)$$

where $V_X = \ker[C_y \ D_{y\theta} \ D_{yp}]$, $V_Y = \ker[B_u^\top \ D_{\theta y}^\top \ D_{pu}^\top]$ and $\Gamma = \mathrm{diag}(1/\gamma I, -\gamma I)$ such that $\gamma > 0$, which represents the induced $\mathcal{L}_2$-gain as the desired performance measure.

The multipliers M and N are related to the LPV scheduling channels, which can be partitioned as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^\top & M_{22} \end{bmatrix}, \quad N = \begin{bmatrix} N_{11} & N_{12} \\ N_{12}^\top & N_{22} \end{bmatrix}. \quad (19)$$

It is desired here to synthesize an affinely scheduled controller such that the scheduling block Θ of the plant is copied to the controller; hence, we consider the following conditions for the multipliers (19) (Hoffmann et al., 2014), $M_{11} > 0$, $M_{11} = -M_{22}$, $M_{12} = -M_{12}^\top$, $N_{11} > 0$, $N_{11} = -N_{22}$, $N_{12} = -N_{12}^\top$, $M_{ij}\Theta = \Theta M_{ij}$, $N_{ij}\Theta = \Theta N_{ij}$, $i, j \in \{1, 2\}$. These constraints are referred to as D/G-scaling. The construction of the extended matrices X_{cl} and multipliers M_{cl} are necessary to solve for the controller matrices in LMI problems, which follows along the lines of (Dettori and Scherer, 2001). Furthermore, when solving for the controller matrices, $D_{\theta\theta}^K = 0$ has to be imposed in order to enforce affine scheduling. The parameter-dependent state-space model matrices of the affinely scheduled controller $K(\theta)$ are then computed by
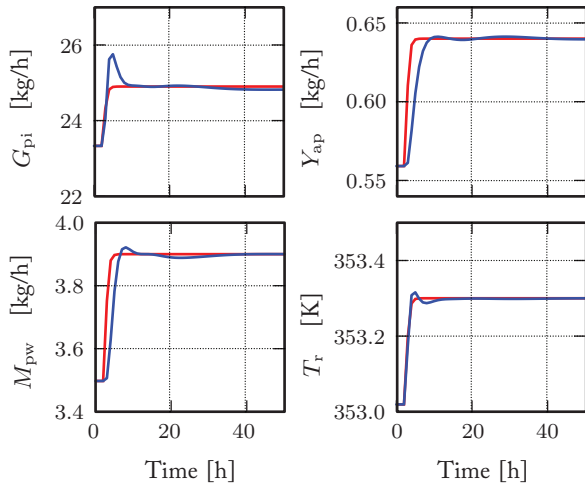
Fig. 3. Closed-loop response for the transition from OP1 to OP2 with the LPV controller, reference signal (red).

$$\begin{bmatrix} \mathcal{A}_K(\theta) & \mathcal{B}_K(\theta) \\ \mathcal{C}_K(\theta) & \mathcal{D}_K(\theta) \end{bmatrix} = \begin{bmatrix} A^K & B_u^K \\ C_y^K & D_{yu}^K \end{bmatrix} + \begin{bmatrix} B_\theta^K \\ D_{y\theta}^K \end{bmatrix} \Theta \begin{bmatrix} C_\theta^K & D_{\theta u}^K \end{bmatrix}, \quad (20)$$

which is a 10th order 3×3 LPV controller for the problem under study. In terms of the control implementation, the LPV controller (20) requires relatively low online computation as it is just required to update the controller state-space matrices at each time instant given the value of the Θ block.

4.2 LPV Controller Implementation

The implementation of the LPV controller on the full NL model of the plant is shown in Fig. 3 and Fig. 4, which illustrate the closed-loop output and input responses, during the transition from OP1 to OP2. Note that, to compute the state-space matrices of the controller via (20) at each sampling instant, the measurement/estimation of p should be available to compute θ and consequently to construct the block $\Theta \in \mathbb{R}^{4 \times 4}$. As observed in (Özkana et al., 2003), the production rate G_{pi} and the temperature T_{r} show faster response in comparison with the polymer composition Y_{ap} and the molecular weight M_{pw}; however, all outputs require less than 10 hours to reach the steady-state values without violating the input constraints. This provides faster response than the controller proposed in (Özkana et al., 2003), which was more than 15 hours. This has a significant impact on reducing the off-specification products.

The effect of an unmeasured disturbance is examined next. We study the effect of the presence of an inhibitor flow in the fresh feed during the transition from OP1 to OP2, i.e., $F_z \neq 0$. The capability of the LPV controller is demonstrated for an inhibitor disturbance of 4 parts per 1000 (mole basis) during the period (1.5-3.0 h) as in (Özkana et al., 2003). In contrast with the MPC controller examined in (Özkana et al., 2003), the LPV controller here rejects the disturbance effect without yielding any of the input flow rates to be saturated and without showing aggressive response as shown in Fig. 5 and Fig. 6. Moreover, the control inputs almost stay within the same ranges as without disturbance, compare Figs. 4 and 6. For the tracking capability, some oscillations are observed
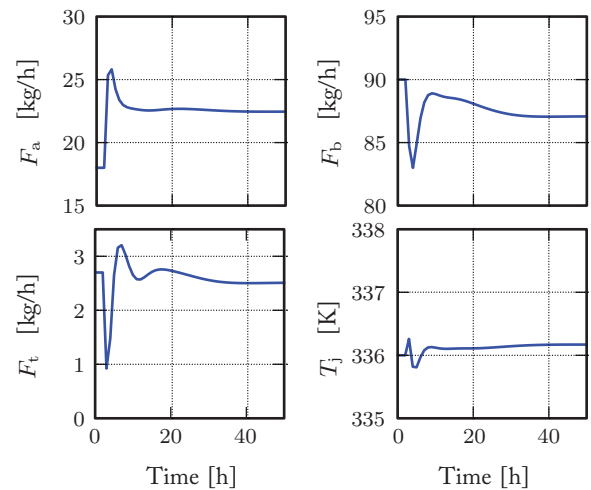

Fig. 4. Manipulated variables during the transition from OP1 to OP2 with the LPV controller.
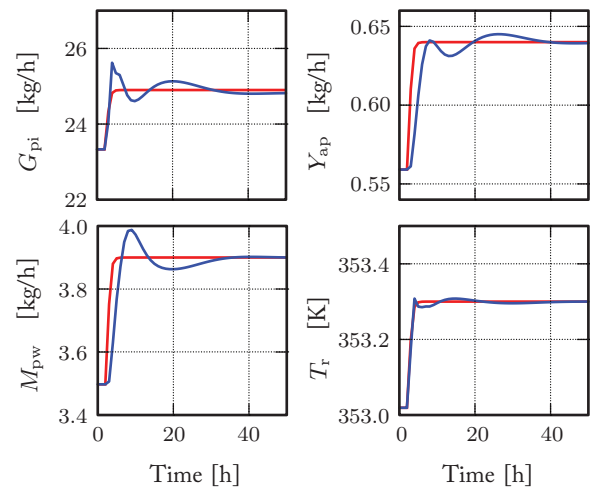

Fig. 5. Closed-loop response in the presence of an unmeasured disturbance in inhibitor during the transition from OP1 to OP2, reference signal (red).

in the outputs, see Fig. 5, that vanish within less than 30 hours to reach the desired values, which is still faster than the response with the controller proposed in (Özkana et al., 2003). A small overshoot is also observed in the M_{pw} response.

It should be emphasized that in contrast with the model predictive control implementation in (Özkana et al., 2003), which was based on multiple piecewise linear models of the process, the LPV controller of this paper is designed based on an exact representation of the original NL model, and hence, stability and performance of the closed-loop system are guaranteed.

5. CONCLUSIONS

In this paper, a low complexity LPV model for a solution copolymerization reactor has been derived by truncating the system states that do not explicitly appear in the output equations of the model. In order to preserve the same input-output behavior as the full model, the truncated states of the reduced model are considered as external scheduling signals. Based on developed reduced-order
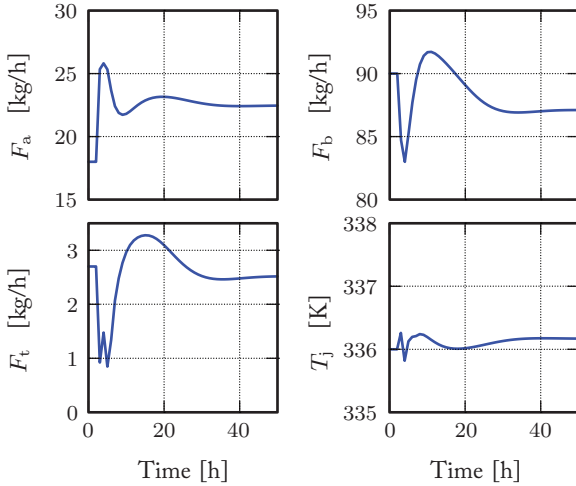
Fig. 6. Manipulated variables response in the presence of an unmeasured disturbance in inhibitor during the transition from OP1 to OP2.

model, a standard gain-scheduled LPV controller has been designed and successfully implemented on the full model to ensure the closed-loop system stability and performance. A satisfactory performance of the process has been achieved with the LPV controller when transitioning between two operating points with and without input disturbance and the results have been compared with previously reported ones.

ACKNOWLEDGEMENTS

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Appendix A. NONLINEAR FUNCTIONS

The NL functions in model (1) are as follows:

$$\phi_l = f_l(C_a, C_b, C_i, C_z, T_r) = \frac{R_k}{C_k},$$

$$\phi_7 = f_7(C_a, C_b, C_i, C_z, T_r) = (-\Delta H_{paa})k_{paa}C_a^{\cdot},$$

$$\phi_8 = f_8(C_a, C_b, C_i, C_z, T_r) = (-\Delta H_{pba})k_{pba}C_a C_b^{\cdot}/C_b$$
$$+ (-\Delta H_{pab})k_{pab}C_a^{\cdot} + (-\Delta H_{pbb})k_{pbb}C_b^{\cdot},$$

$$\phi_9 = f_9(C_a, C_b, C_i, C_s, C_t, C_z, T_r) = \big(k_{caa}(\psi_0^{a\cdot})^2$$
$$+ k_{cab}\psi_0^{a\cdot}\psi_0^{b\cdot} + L_1\psi_0^{a\cdot}\big)/2C_a,$$

$$\phi_{10} = f_{10}(C_a, C_b, C_i, C_s, C_t, C_z, T_r) = \big(k_{cbb}(\psi_0^{b\cdot})^2$$
$$+ L_2\psi_0^{b\cdot}\big)/2C_b,$$

$$\phi_{11} = f_{11}(C_a, C_b, C_i, C_s, C_t, C_z, T_r) = \big(k_{caa}\psi_0^{a\cdot}\psi_1^{a\cdot}$$
$$+ k_{cab}(\psi_0^{b\cdot}\psi_1^{a\cdot}) + L_1\psi_1^{a\cdot}\big)/C_a$$

$$\phi_{12} = f_{12}(C_a, C_b, C_i, C_s, C_t, C_z, T_r) = \big(k_{cab}(\psi_0^{a\cdot}\psi_1^{b\cdot})$$
$$+ k_{cbb}\psi_0^{b\cdot}\psi_1^{b\cdot} + L_2\psi_1^{b\cdot}\big)/C_b$$

$$\phi_{13} = f_{13}(C_a, C_b, C_i, C_s, C_t, C_z, T_r) = \big(k_{caa}((\psi_1^{a\cdot})^2$$
$$+ \psi_0^{a\cdot}\psi_2^{a\cdot}) + k_{cab}(2\psi_1^{a\cdot}\psi_1^{b\cdot} + \psi_2^{b\cdot}\psi_0^{a\cdot}) + L_1\psi_2^{a\cdot}\big)/C_a$$

$$\phi_{14} = f_{14}(C_a, C_b, C_i, C_s, C_t, C_z, T_r) = \big(k_{cab}(\psi_2^{a\cdot}\psi_0^{b\cdot})$$
$$+ k_{cbb}\big((\psi_1^{b\cdot})^2 + \psi_0^{b\cdot}\psi_2^{b\cdot}\big) + L_2\psi_2^{b\cdot}\big)/C_b$$

$$\phi_{15} = f_{15}(\lambda_a, \lambda_b) = 1/(\lambda_a + \lambda_b),$$

$$\phi_{16} = f_{16}(\psi_1) = 1/\psi_1,$$

where R_k's are the reaction rates, which are given by equations (4-9) in (Congalidis et al., 1989); $C_a^{\cdot}$, $C_b^{\cdot}$, $\psi_0^{a\cdot}$, $\psi_1^{a\cdot}$, $\psi_2^{a\cdot}$, $\psi_0^{b\cdot}$, $\psi_1^{b\cdot}$ and $\psi_2^{b\cdot}$ are defined in equations (11), (12), (31-36), respectively, in (Congalidis et al., 1989), and $-\Delta H_{paa}$, $-\Delta H_{pba}$, $-\Delta H_{pab}$ and $-\Delta H_{pbb}$ are thermodynamic constant parameters.