

A Kernel-based Approach to MIMO LPV State-space Identification and Application to a Nonlinear Process System ^{*}

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Abstract: This paper first describes the development of a nonparametric identification method for linear parameter-varying (LPV) state-space models and then applies it to a nonlinear process system. The proposed method uses kernel-based least-squares support vector machines (LS-SVM). While parametric identification methods require proper selection of basis functions in order to avoid over-parametrization or structural bias, the problem of variance-bias tradeoff is avoided by estimating the functional dependencies of the state-space representation on the LPV scheduling variables using measured input and output data under the LS-SVM framework. The proposed formulation allows for LS-SVM to reconstruct and uncover static, as well as dynamic dependencies on scheduling variables in multi-input multi-output (MIMO) LPV models. This is achieved by assuming that the states are measurable, which is a common scenario during online control of many chemical processes described by lumped parameter models. The proposed method does not require an explicit declaration of the feature maps of the nonlinearities of the assumed model structure; instead, it requires the selection of a nonlinear kernel function and tuning its parameters. The developed identification method is applied to a continuous stirred tank reactor (CSTR) model under realistic noise conditions. Another numerical example along with the CSTR system illustrates the performance of the proposed algorithm under both static and dynamic dependence on the scheduling variables.

Keywords: Nonparametric identification; LPV systems; Kernel functions; LS-SVM; process systems

1. INTRODUCTION

Linear parameter-varying (LPV) models provide a powerful framework for identification of nonlinear systems. Under this framework, nonlinear models can be represented as a linear dynamic relation of the input and output variables; the relation is itself dependent on measurable time-varying signals, commonly known as *scheduling variables*. These scheduling signals express the varying operating conditions of the system. Thus, LPV models represent an intermediate stage between linear time-invariant (LTI) and nonlinear systems, while preserving many attractive attributes of LTI systems. This simplicity of LPV models allows one to apply linear optimal control techniques to nonlinear systems represented by LPV models, opening up the possibility of applying powerful LPV control synthesis tools. In particular, in process systems, it has often been observed by control engineers and modeling practitioners that the dynamics of the process is well captured by linear models at any given operating condition. The concept of LPV

models, therefore, comes in very handy in order to extend the LTI models over a wide range of varying operating conditions.

As a natural consequence of this property, LPV identification has attracted a lot of attention in the past decade (Rugh and Shamma (2000)), with different identification schemes developed for both LPV input-output and state-space models (Verdult and Verhaegen (2005); Felici et al. (2007); Tóth et al. (2011); Lee and Poolla (1999); Bamieh and Giarre (2002); Verdult (2002); Casella and Lovera (2008)).

For LPV state-space model identification, to the best of authors' knowledge, most techniques in the literature fall under the category of parametric approaches. In parametric approaches, the scheduling dependencies of the model coefficients are described as a linear combination of basis functions that need to be chosen *a priori*. However, the selection of these basis functions remains difficult since overestimating the number of basis functions leads to over-parameterized models and hence a large variance in the estimates despite the low order of the model. On the other hand, an inappropriate selection of these functions is known to cause structural bias (see Tóth et al. (2011)). For state-space LPV and bilinear models, parametric methods are

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based on various subspace approaches, which are extensions of the well-accepted subspace identification methods used for LTI systems. These methods usually require a high computational demand due to the enormous dimension of the data matrices involved (the growth is polynomial in the state dimension and exponential in the scheduling variables). Verhaegen et al. proposed a solution to overcome this curse of dimensionality on the expense of approximation of the data equations and identify LPV state-space models with affine parameter dependence (Verdult and Verhaegen (2002)). Other subspace-based methods were later published in Felici et al. (2007); Wingerden and Verhaegen (2008); Tanelli et al. (2011); Lovera and Mercère (2007).

Nonparametric methods provide an alternative way to avoid the bias-variance tradeoff by obtaining nonparametric reconstruction of the scheduling dependencies in LPV models. In particular, with the emergence of kernel-based techniques, a new avenue of nonparametric identification, classification, and data processing has appeared in the last two decades. Kernels are functions that enable us to perform linear operations in high-dimensional feature spaces, often mapping nonlinear dependencies very efficiently using the so-called *kernel trick* (Schölkopf and Smola (2002)). This has sprouted the use of kernel-based techniques for solving different problems under the umbrella of LPV system identification, ranging from LPV model reduction (Rizvi et al. (2014)) to estimating coefficient dependencies in LPV I/O models (Tóth et al. (2011)). Verhaegen et al. incorporated kernel methods in their earlier subspace-based technique in order to reduce high dimensional data matrices (see Verdult and Verhaegen (2005)). The identification approaches published in Tóth et al. (2011); Abbasi et al. (2014); Golabi et al. (2014) reported efficient kernel-based methods employing LS-SVM for LPV I/O models; the results showed consistent estimates of the coefficient dependencies with a very attractive bias-variance tradeoff. A mixed parametric method for LPV state-space identification was proposed recently in dos Santos et al. (2014). The authors described the C matrix using a nonparametric LS-SVM-based model, while the A matrix was described by a parametric model. The model structure was assumed to be in companion reachability canonical form (CR-CF), and the coefficients were estimated using an iterative routine.

In this work, we specifically aim to use the properties of LS-SVM for LPV modeling of nonlinear process systems. Many chemical processes, including high purity distillation columns, exothermic and non-isothermal chemical reactors, and batch systems are inherently nonlinear. Due to their nonlinearities, they cannot be efficiently stabilized and monitored with controllers and estimators designed on the basis of linearized models around an operating point. Sources ranging from the Arrhenius temperature dependence of reaction rates, radiative heat transfer phenomena, to complex reaction mechanisms cause for the highly nonlinear behaviors in chemical processes (Christofides and El-Farra (2005)). Increasingly, limitations of traditional linear control and modeling methods have become apparent in dealing with nonlinear chemical processes as processes are required to operate over a wide range of conditions. *Multi-input multi-output* (MIMO) LPV state-space models can fill this gap by modeling the processes with a linear dynamic relation between the process inputs and outputs, where the relation itself is a nonlinear function of time-varying parameters.

This way, several linear control techniques can be easily applied to systems represented by an LPV model.

In this paper, we present a nonparametric kernel-based identification method for MIMO LPV state-space models with measurable states; this is a common situation in several lumped parameter models of process systems in which the states are directly accessible for measurement. We employ LS-SVM in order to explore the coefficient dependencies on the scheduling variables with a good variance-bias tradeoff. We finally validate the proposed technique on the model of an ideal continuous stirred tank reactor (CSTR). The paper is arranged as follows. The problem formulation is presented in Section 2. The LPV state-space model is formulated in an LS-SVM setting and identification algorithm is derived in Section 3. Numerical examples are provided in Section 4, where a discussion about the results is also given. Concluding remarks are finally made in Section 5.

2. PROBLEM FORMULATION

Consider an LPV system represented by the following discrete-time state-space model with innovation noise model

$$\begin{aligned} x_{k+1} &= A(p_k)x_k + B(p_k)u_k + K(p_k)e_k, \\ y_k &= C(p_k)x_k + e_k, \end{aligned} \quad (1)$$

where k denotes discrete time, matrices $A(p_k) \in \mathbb{R}^{n \times n}$, $B(p_k) \in \mathbb{R}^{n \times n_u}$, $K(p_k) \in \mathbb{R}^{n \times n_y}$, and $C(p_k) \in \mathbb{R}^{n_y \times n}$ are functions of time-varying scheduling variables $p_k \in \mathbb{R}^{n_p}$, and $e_k \in \mathbb{R}^{n_y}$ is a stochastic white noise process. In addition, x and y represent the model states and sensor measurements, respectively. Assuming that the states are available for measurement, we aim at employing nonlinear kernel functions under the LS-SVM framework in order to estimate the functional dependencies of the state-space matrices on the scheduling variables. We can rewrite (1) as

$$\begin{aligned} x_{k+1} &= \underbrace{(A(p_k) - K(p_k)C(p_k))}_{\tilde{A}(p_k)} x_k + B(p_k)u_k + K(p_k)y_k, \\ y_k &= C(p_k)x_k + e_k, \end{aligned} \quad (2)$$

which can be reformulated as follows

$$\begin{aligned} x_{k+1} &= W_1 \Phi_1(p_k)x_k + W_2 \Phi_2(p_k)u_k + W_3 \Phi_3(p_k)y_k, \\ y_k &= W_4 \Phi_4(p_k)x_k + e_k, \end{aligned} \quad (3)$$

where $W_{1,2,3} \in \mathbb{R}^{n \times n_H}$ and $W_4 \in \mathbb{R}^{n_y \times n_H}$ are unknown weighting matrices, while matrices $\Phi_1(p_k) \in \mathbb{R}^{n_H \times n}$, $\Phi_2(p_k) \in \mathbb{R}^{n_H \times n_u}$, $\Phi_3(p_k) \in \mathbb{R}^{n_H \times n_y}$, and $\Phi_4(p_k) \in \mathbb{R}^{n_H \times n}$ represent unknown feature maps. Variable n_H represents dimension of, a possibly infinite dimensional feature space. The problem, therefore, boils down to finding the state-space matrices dependencies $W_i \Phi_i(p_k)$ for $i = 1, \dots, 4$, given the data $\{u_k, y_k, x_k, p_k\}_{k=1}^N$, where N is the number of data points (samples).

3. KERNEL-BASED LPV STATE-SPACE MODEL IDENTIFICATION

The estimates of the LPV state-space matrices in the form of $W_i \Phi_i(p_k)$ can be obtained by minimizing the following cost function

$$J = \frac{1}{2} \sum_{i=1}^4 \|W_i\|_F^2 + \frac{1}{2} \sum_{k=1}^N e_k^\top \Gamma e_k, \quad (4)$$

over $W_{1,2,3,4}$, where $\|\cdot\|_F$ denotes the Frobenius norm, and $\Gamma = \text{diag}(\gamma_1, \dots, \gamma_n)$ is a diagonal weighting matrix on the residual errors e_k , and is known as the regularization matrix. The afore-described optimization problem can be solved by introducing

Lagrange multipliers and substituting the inner product $\Phi_i \Phi_i^\top$ using an *a priori* chosen nonlinear kernel function as shown in Tóth et al. (2011). This is expected to give us a nonparametric estimate of the coefficients function matrices representing the state-space matrices of the original LPV model. We define the Lagrangian function as

$$\begin{aligned} \mathcal{L}(W_1, W_2, W_3, W_4, \alpha, \beta, e) = \\ J - \sum_{j=1}^N \alpha_j^\top \{W_1 \Phi_1(p_j) x_j + W_2 \Phi_2(p_j) u_j + W_3 \Phi_3(p_j) y_j - x_{j+1}\} \\ - \sum_{j=1}^N \beta_j^\top \{W_4 \Phi_4(p_j) x_j + e_j - y_j\}, \end{aligned} \quad (5)$$

where $\alpha_j \in \mathbb{R}^n$, $\beta_j \in \mathbb{R}^{n_y}$ are the Lagrange multipliers at discrete time j . Due to the convexity of the problem, the global optimum is obtained when the derivatives are equal to zero as follows

$$\frac{\partial \mathcal{L}}{\partial \alpha_j} = 0 \Rightarrow x_{j+1} = W_1 \Phi_1(p_j) x_j + W_2 \Phi_2(p_j) u_j + W_3 \Phi_3(p_j) y_j, \quad (6a)$$

$$\frac{\partial \mathcal{L}}{\partial W_1} = 0 \Rightarrow W_1 = \sum_{j=1}^N \alpha_j x_j^\top \Phi_1^\top(p_j), \quad (6b)$$

$$\frac{\partial \mathcal{L}}{\partial W_2} = 0 \Rightarrow W_2 = \sum_{j=1}^N \alpha_j u_j^\top \Phi_2^\top(p_j), \quad (6c)$$

$$\frac{\partial \mathcal{L}}{\partial W_3} = 0 \Rightarrow W_3 = \sum_{j=1}^N \alpha_j y_j^\top \Phi_3^\top(p_j), \quad (6d)$$

$$\frac{\partial \mathcal{L}}{\partial W_4} = 0 \Rightarrow W_4 = \sum_{j=1}^N \beta_j x_j^\top \Phi_4^\top(p_j), \quad (6e)$$

$$\frac{\partial \mathcal{L}}{\partial e_j} = 0 \Rightarrow \beta_j = \Gamma e_j, \quad (6f)$$

$$\frac{\partial \mathcal{L}}{\partial \beta_j} = 0 \Rightarrow y_j = W_4 \Phi_4(p_j) x_j + e_j. \quad (6g)$$

Substituting (6a)-(6g) in (3), we can write the following

$$\begin{aligned} x_{k+1} &= W_1 \Phi_1(p_k) x_k + W_2 \Phi_2(p_k) u_k + W_3 \Phi_3(p_k) y_k \\ &= \underbrace{\left\{ \sum_{j=1}^N \alpha_j x_j^\top \Phi_1^\top(p_j) \right\}}_{W_1} \Phi_1(p_k) x_k + \underbrace{\left\{ \sum_{j=1}^N \alpha_j u_j^\top \Phi_2^\top(p_j) \right\}}_{W_2} \Phi_2(p_k) u_k \\ &\quad + \underbrace{\left\{ \sum_{j=1}^N \alpha_j y_j^\top \Phi_3^\top(p_j) \right\}}_{W_3} \Phi_3(p_k) y_k, \quad (7) \\ y_k &= \underbrace{\left\{ \sum_{j=1}^N \beta_j x_j^\top \Phi_4^\top(p_j) \right\}}_{W_4} \Phi_4(p_k) x_k + \underbrace{\Gamma^{-1} \beta_k}_{e_k}. \quad (8) \end{aligned}$$

Replacing the inner-product $\Phi_i(p_k)^\top \Phi_i(p_j)$ by a kernel function $\bar{k}^i(p_j, p_k)$, we further define kernel matrices Ω and Ξ as

$$\begin{aligned} [\Omega]_{j,k} &= \sum_{i=1}^3 z_i^\top(j) \bar{k}^i(p_j, p_k) z_i(k), \\ [\Xi]_{j,k} &= x_j^\top \bar{k}^4(p_j, p_k) x_k, \end{aligned} \quad (9)$$

where
$$z_i(k) = \begin{cases} x_k, & i = 1 \\ u_k, & i = 2 \\ y_k, & i = 3. \end{cases}$$

While a wide variety of kernel functions exists in the literature to choose from, commonly used kernels include the *Radial*

Basis Function (RBF), *polynomial* or *sigmoid* kernels among many others (see Schölkopf and Smola (2002)). A typical RBF kernel, also known as the Gaussian kernel, is represented by

$$\bar{k}^i(p_j, p_k) = \exp\left(-\frac{\|p_j - p_k\|_2^2}{2\sigma_i^2}\right), \quad (10)$$

where σ_i is a free kernel parameter, and $\|\cdot\|_2$ represents the 2-norm. We can now write (7)-(8) in a compact form as follows

$$X_{k+1} = \alpha \Omega, \quad (11)$$

$$Y = \beta \Xi + \Gamma^{-1} \beta, \quad (12)$$

where $\Omega \in \mathbb{R}^{N \times N}$ and $\Xi \in \mathbb{R}^{N \times N}$ are kernel matrices as defined above, $\alpha = [\alpha_1 \cdots \alpha_N] \in \mathbb{R}^{n \times N}$ and $\beta = [\beta_1 \cdots \beta_N] \in \mathbb{R}^{n_y \times N}$ are the matrices containing the Lagrange multipliers, $X_{k+1} = [x_2 \cdots x_{N+1}] \in \mathbb{R}^{n \times N}$ and $Y = [y_1 \cdots y_N] \in \mathbb{R}^{n_y \times N}$ contain the states and outputs for the N samples, respectively. The solution to the above equations can be obtained as follows

$$\alpha = X_{k+1} \Omega^{-1}, \quad (13)$$

$$\text{vec}(\beta) = \left(I_N \otimes \Gamma^{-1} + \Xi^\top \otimes I_{n_y} \right)^{-1} \text{vec}(Y), \quad (14)$$

where \otimes denotes the Kronecker product and $\text{vec}(\cdot)$ denotes vectorization function, which stacks subsequent columns in a matrix below one another; matrices I_N and I_{n_y} represent identity matrices of dimensions N and n_y , respectively. The solution (14) is obtained using the solution to the classical sylvester equation (Bartels and Stewart (1972)). Once trained, the estimate of the state-space matrices can be calculated by using (6b)-(6e) as

$$\tilde{A}_e(\cdot) = W_1 \Phi_1(\cdot) = \sum_{k=1}^N \alpha_k x_k^\top \bar{k}^1(p_k, \cdot), \quad (15a)$$

$$B_e(\cdot) = W_2 \Phi_2(\cdot) = \sum_{k=1}^N \alpha_k u_k^\top \bar{k}^2(p_k, \cdot), \quad (15b)$$

$$K_e(\cdot) = W_3 \Phi_3(\cdot) = \sum_{k=1}^N \alpha_k y_k^\top \bar{k}^3(p_k, \cdot), \quad (15c)$$

$$C_e(\cdot) = W_4 \Phi_4(\cdot) = \sum_{k=1}^N \beta_k x_k^\top \bar{k}^4(p_k, \cdot), \quad (15d)$$

where subscript e denotes estimate. Once estimates of \tilde{A}, C, K are obtained, estimate $A_e = \tilde{A}_e + K_e C_e$ can be calculated accordingly. This gives a nonparametric estimate of the state-space matrices. It is noteworthy here that the parameter matrices W_i or the basis functions $\Phi_i(\cdot)$ are not accessible explicitly. What we are able to estimate via nonlinear kernel functions is $W_i \Phi_i(\cdot)$.

Identification of LPV models with dynamic dependence on the scheduling variables

Next, we consider the case, where the state-space matrices of the LPV model have dynamic dependence on the scheduling variables. Such an LPV state-space model can be described by

$$\begin{aligned} x_{k+1} &= A(p_k, \tilde{n}) x_k + B(p_k, \tilde{n}) u_k + K(p_k, \tilde{n}) e_k, \\ y_k &= C(p_k, \tilde{n}) x_k + e_k, \end{aligned} \quad (16)$$

where $A(p_k, \tilde{n}) = A(p_k, \cdots, p_{k-\tilde{n}})$ signifies the dependence of A on \tilde{n} past values of the scheduling variables. Substituting $e_k = y_k - C(p_k, \tilde{n}) x_k$ in the equation for x_{k+1} , we can rewrite the above set of equations as

$$\begin{aligned} x_{k+1} &= \tilde{A}(p_k, \tilde{n}) x_k + B(p_k, \tilde{n}) u_k + K(p_k, \tilde{n}) y_k, \\ y_k &= C(p_k, \tilde{n}) x_k + e_k, \end{aligned} \quad (17)$$

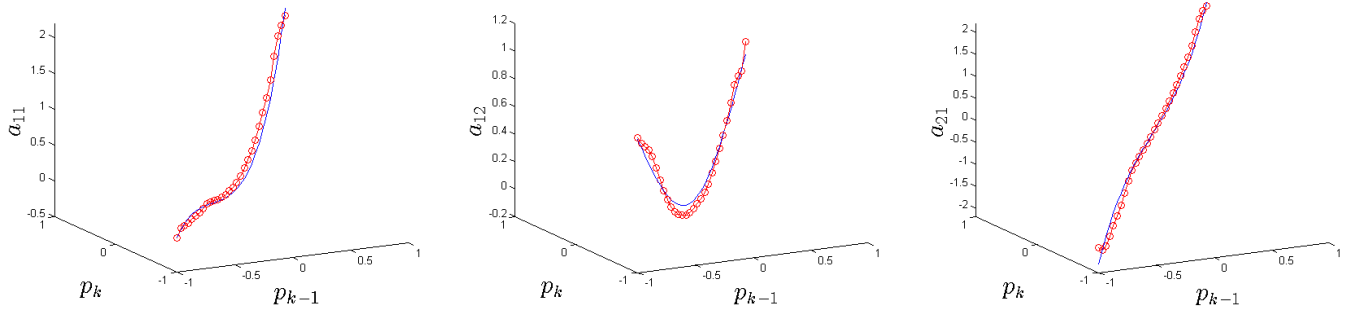


Fig. 1. Example 1: Elements (functions) a_{11} , a_{12} and a_{21} of the state matrix as a function of scheduling variable p_k and its delayed sample p_{k-1} . Solid blue line represents the original elements while the circled red line indicates their estimates.

where $\tilde{A}(p_k, \tilde{n}) = A(p_k, \tilde{n}) - K(p_k, \tilde{n})C(p_k, \tilde{n})$. Following the same procedure as before, we arrive at the following equation for the states x_k for $k \in \{1, \dots, N\}$

$$x_{k+1} = \sum_{i=1}^3 \left\{ \left(\sum_{j=1}^N \alpha_j z_i^\top(j) \Phi_i^\top(p_j, \tilde{n}) \right) \Phi_i(p_k, \tilde{n}) z_i(k) \right\},$$

$$y_k = \left\{ \sum_{j=1}^N \beta_j x_j^\top \Phi_4^\top(p_j, \tilde{n}) \right\} \Phi_4(p_k, \tilde{n}) x_k + \Gamma^{-1} \beta_k, \quad (18)$$

where $z_i(k)$ is defined as before. The kernel matrices Ω and Ξ can then be written in a modified form as

$$[\Omega]_{j,k} = \sum_{i=1}^3 z_i^\top(j) \bar{k}^i(p(j, \tilde{n}), p(k, \tilde{n})) z_i(k).$$

$$[\Xi]_{j,k} = x_j^\top \bar{k}^4(p(j, \tilde{n}), p(k, \tilde{n})) x_k. \quad (19)$$

The RBF kernel is calculated as follows

$$\bar{k}^i(p(j, \tilde{n}), p(k, \tilde{n})) = \exp\left(-\frac{\|\mathbf{p}(j, \tilde{n}) - \mathbf{p}(k, \tilde{n})\|_2^2}{2\sigma_i^2}\right), \quad (20)$$

where $\mathbf{p}(j, \tilde{n}) = [p_j^\top \ p_{j-1}^\top \ \dots \ p_{j-\tilde{n}}^\top]^\top$. Other kernels can be defined in a similar way. By applying the kernel function over a dynamic range of present and past values of the scheduling variables, dynamic coefficient dependencies of the state-space matrices on the scheduling variables can be mapped. In the next section, we examine the performance of the developed algorithm by means of different nonlinear examples with static and dynamic dependence on the scheduling variables.

4. NUMERICAL EXAMPLES

4.1 Example 1

The following numerical example of a second order discrete-time LPV state-space model is considered.

$$x_{k+1} = A(p_k, p_{k-1})x_k + B(p_k)u_k + e_k,$$

with

$$A(p_k, p_{k-1}) = \begin{bmatrix} p_k^3 + p_{k-1}^2 & p_k \tanh(p_{k-1}) \\ p_k^3 + p_{k-1} & p_k^2 \end{bmatrix}, \text{ and}$$

$$B(p_k) = \begin{bmatrix} \text{sat}(p_k) \\ \sin(2\pi p_k) + \cos(2\pi p_k) \end{bmatrix},$$

where

$$\text{sat}(p_k) = \begin{cases} -0.5, & p_k < -0.5 \\ 0.5, & p_k > 0.5 \\ 0 & \text{otherwise.} \end{cases}$$

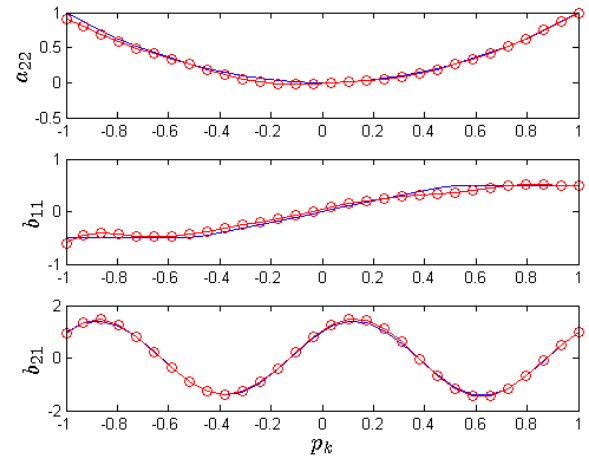


Fig. 2. Example 1: Elements a_{22} , b_{11} and b_{21} of the LPV state-space matrices as a function of the scheduling variable p_k . The solid blue line represents the original functions while the circled red line indicates their estimates.

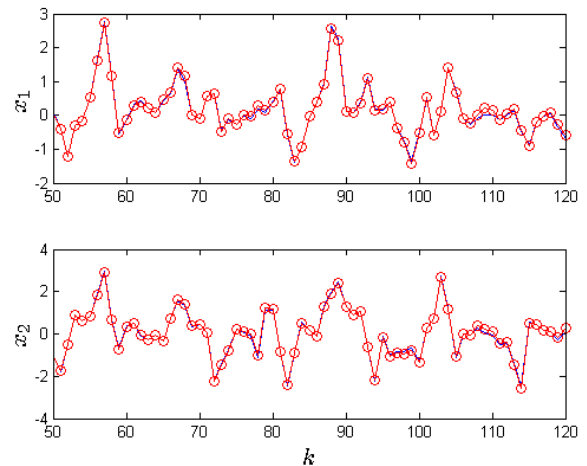


Fig. 3. Example 1: States x_1 , x_2 of the LPV system model. The solid blue and the circled red lines represent the original and simulated state response of the estimated model, respectively. Variable k denotes time samples.

Table 1. Example 1: Monte-Carlo simulation results for example 1: BFR values for the underlying coefficient functions.

Fcn	Mean (BFR)	STD (BFR)	Fcn	Mean (BFR)	STD (BFR)
a_{11}	89.24%	0.532	a_{12}	88.01%	0.718
a_{21}	93.01%	0.098	a_{22}	92.05%	0.188
b_{11}	87.22%	1.102	b_{21}	92.54%	0.021

As can be noticed, elements a_{11}, a_{12} and a_{21} have a dynamic dependence on p_k , while a_{22}, b_{11} and b_{21} depend only on p_k . A total of 1200 samples of scheduling variables $p_k \in [-1, 1]$ are generated as $p_k = \sin(0.5k)$. Input signals u_k are generated randomly. The generated data is divided into 900 and 300 samples for training and validation, respectively. Gaussian white noise e_k is added such that an output signal-to-noise ratio (SNR) of 25 dB is maintained. RBF kernel is chosen with its parameters tuned as $\sigma_i = 0.45 \forall i$ and $\Gamma = \text{diag}\{300, 300\}$. The proposed LS-SVM algorithm is run and the Lagrange multipliers are estimated. Validation is performed on the validation data; we define *Best Fit Ratio* (BFR) as

$$\text{BFR} := 100\% \cdot \max \left(\frac{\|x - \hat{x}\|_2}{\|x - \bar{x}\|_2}, 0 \right),$$

where \hat{x} and \bar{x} represent predicted states and mean value of the states, respectively. An average output BFR of 92.05% with a standard deviation of 1.54 is achieved for 100 runs of the Monte-Carlo simulation. BFR statistical information for the underlying functional dependencies of the coefficients over $p \in [-1, 1]$ is tabulated in Table 1. Estimated elements a_{11}, a_{12} , and a_{21} of the state-space matrix A that have dynamic dependence on the scheduling variable are shown in Figure 1. Other functions, namely a_{22}, b_{11} and b_{21} are shown in Figure 2; the estimated functions do not show dependency on time-shifted scheduling variables, and are hence, plotted in a two-dimensional plot. Figure 3 shows predicted states as compared to the actual ones. The BFR values, as well as the approximated functional dependencies shown in the figures, demonstrate the remarkable ability of the proposed kernel-based method to estimate nonlinear dependencies, with both static and dynamic dependence, with a great accuracy.

4.2 Example 2: A continuous stirred tank reactor

In the second example, the model of an ideal *continuous stirred tank reactor* (CSTR) is considered. Schematic diagram of the CSTR process is shown in Figure 4. It shows the chemical reaction, under ideal conditions, that converts an inflowing liquid to a product; this reaction is non-isothermal as described in Tóth et al. (2010). A heat coolant-based exchanger is used in order to control the temperature inside the reactor. The first principles-based model is described as

$$\begin{aligned} \dot{C}_2 &= \frac{Q_1}{V} (C_1 - C_2) - k_0 e^{-\frac{E_A}{RT_2}} C_2, \\ \dot{T}_2 &= \frac{Q_1}{V} (T_1 - T_2) - \frac{U_{HE}}{A_{HE}} (T_2 - T_c) + \frac{\Delta H k_0}{\rho c_p} e^{-\frac{E_A}{RT_2}} C_2. \end{aligned} \quad (21)$$

A typical control objective is to regulate the concentration and the temperature in the reactor, denoted by C_2 and T_2 , respectively. To this end, variables Q_1 and T_c , which represent the flow of inflowing liquid and the temperature of the coolant, respectively, are used as manipulatable control signals. Steady-state operating conditions as described in Roffel and Betlem (2007) and Tóth et al. (2010), are tabulated in Table 2. The

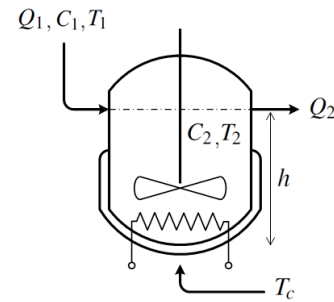


Fig. 4. An ideal continuous stirred tank reactor

Table 2. Example 2: Steady-state values of variables and constants for the CSTR model

V	Reactor volume	5 m^3
C_1	Concentration of the inflowing liquid	800 kg/m^3
C_2	Concentration in the reactor	213.69 kg/m^3
Q_1	Inflowing rate	$0.01 \text{ m}^3/\text{s}$
Q_2	Outflowing rate	$0.01 \text{ m}^3/\text{s}$
k_0	Pre-exponential term	25 s^{-1}
E_A	Activation energy of reaction	$30,000 \text{ (J/kg)}$
T_1	Temp. of inflowing liquid	353 K
T_2	Temp. in the reactor	428.5 K
T_c	Coolant temperature	300 K
ρ	Density	800 kg/m^3
c_p	Specific heat	1000 (J/kg.s)
ΔH	Heat of reaction	$125,000 \text{ J/kg}$
U_{HE}	Heat transfer coefficient	$1,000 \text{ (J/kg.s)}$
A_{HE}	Surface area of heat exchanger	1 m^2
h	Liquid level	5 m
R	Gas constant	8.31 (J/mol.K)

authors in Tóth et al. (2010) show that introducing a step-disturbance in the inflowing rate of the liquid shows different behavior in the dynamics of the controlled variables T_2 and C_2 in terms of both the time constant and the relative gain when operating at different values of the inflowing liquid concentration C_1 . Since the raw material can be obtained from different sources, the concentration C_1 can have differing values ranging from 50% to 150% of the nominal value. The dynamics differs not only in the values of the time constant and the relative gains, the response also shows a change in the sign of the gain exhibiting non-minimum phase behavior. It is evident that a PID controller designed for the nominal value of C_1 might easily fail to stabilize the plant. Hence, LPV modeling naturally appears to be a logical representational choice, using the concentration C_1 as the scheduling variable. It is assumed that the reactor is mixed ideally, that the density and the physical properties of the process remain constant, that the reaction is first order with a temperature relation according to Arrhenius law, and that the temperature increase in the coolant over the coil can be neglected. It is also assumed that the inflow and outflow rates, Q_1 and Q_2 , are kept equal to each other.

For the aforementioned system, a *pseudo random binary sequence* (PRBS) of the two inputs are used to excite the CSTR model. A trajectory of slowly varying scheduling variable, C_1 , is generated ranging from 50% to 150% of the nominal value given in Table 2. Gaussian white noise is added such that SNRs of 20dB and 30dB are maintained for T_2 and C_2 , respectively. An RBF kernel is selected for training and regularization, and kernel parameters are tuned to be $\Gamma = \text{diag}\{10^4, 10^4\}$ and $\sigma = 620$. Both σ and Γ are tuned after a fine grid search over possible combinations of parameter values. An adequate sampling

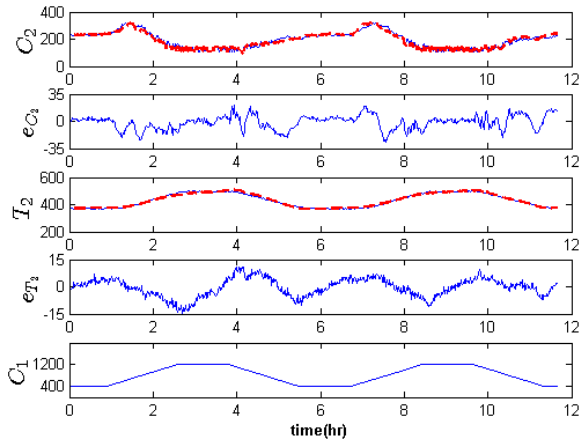


Fig. 5. (Top to bottom): Concentration in reactor C_2 ; error in C_2 estimation; reactor temperature T_2 ; error in T_2 estimation; concentration of inflowing liquid C_1 . Variable C_1 represents the scheduling variable. Solid blue lines represent the actual values while dotted red lines represent the predicted ones.

time of 60s is chosen, and the model is simulated to generate input and output samples. Collected data is divided into training and validation sets and the proposed LS-SVM routine is run. Trained model is then validated on noise-free validation data and results are shown in Figure 5; an output BFR of 85.04% is achieved. The scheduling variable trajectory is also shown in Figure 5. While LPV modeling based on *orthogonal basis functions* (OBF) have shown comparatively higher BFR values for validation data in Tóth et al. (2010), the OBF method uses a polynomial interpolation of order 8. As opposed to that, in the present method, the state order is not increased, and a good approximation is achieved with a second order LPV state-space approximation.

5. CONCLUDING REMARKS

This paper has introduced a non-iterative and nonparametric identification scheme for LPV state-space models whose states are available for measurement, a situation often occurring in process systems, where the states are directly measurable. The proposed technique is an extension to the LS-SVM identification method for LPV models in an input-output form introduced in (Tóth et al. (2011)). We utilize a dual optimization scheme that lowers the variance of the estimates and is able to uncover the structural dependency of a MIMO LPV model without over-parametrization. Both static and dynamic dependence of the state-space matrices on the scheduling variables have been explored under noisy conditions. As a case study, nonlinear model of an ideal CSTR has been considered and the proposed model is used to fit identification data, giving encouraging prediction results when subjected to a fresh set of validation data. The proposed method, which requires only a proper choice of a kernel and tuning of the respective kernel parameters is able to map nonlinear dependencies, thereby providing a model that can be used to design LPV controllers.

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