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Event-triggered Constant Reference Tracking Control for Discrete-time LPV Systems with Application to a Laboratory Tank System

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Abstract: This paper develops a novel event-triggered control design methodology for discrete-time dynamic systems characterized by linear parameter varying (LPV) models. To this end, an event-triggering mechanism is first introduced for LPV systems aiming at reducing the data transmission of states, scheduling variables and controller outputs, i.e., at both the sensor and the controller nodes. Then, an event-based LPV state feedback controller is proposed to achieve a reference tracking objective. Sufficient conditions for simultaneous design of the controller parameters and event-triggering conditions are provided in terms of linear matrix inequality conditions to guarantee the stability of the closed-loop system and to track a desired step reference signal. The experimental results are finally presented to demonstrate and validate the properties and performance of the proposed control design approach on a laboratory tank system.

1 Introduction

Networked control systems have attracted a significant interest recently due to their potential advantages comprising flexibility, low cost installation and maintainability. However, it is worth mentioning that most of the available results in this field are based on the implicit assumption that all signals are sampled and sent periodically with equidistant time intervals, which is called *time-triggering* sampling [1]. Although this type of sampling allows the designer to use the well-developed sampled-data theory and therefore is more preferable from the system analysis and design aspects, it is sometimes less preferable from the resource utilization. Indeed, in the time-triggering sampling, unnecessary information may be sent over communication channels and hence the transmission resources may not be used efficiently. Hence, an *event-triggered* data transmission scheme has been recently introduced to remedy the above issues in the time-triggered scheme. In this approach, a sampling action is triggered only when a given function of the system's state or output exceeds a threshold [2]. Moreover, it is shown that event-triggering control strategies that execute the control task aperiodically when it is needed, can mitigate the unnecessary usage of communication and computational resources and at the same time can guarantee the control objectives [3]. The existing approaches for the design of event-triggered controllers can be classified as belonging to one of the following two categories, namely, emulation-based approaches [4] and co-design based approaches [5, 6]. In the emulation-based framework, the controller is designed without considering the event-triggered nature of the control system, whereas in co-design framework, the feedback controller and the event-triggering strategy are jointly designed [7]. Many important results in event-triggering control design problem are reported in the literature (see, e.g., [7–14]), where most of which are developed for linear time-invariant (LTI) and nonlinear systems and there are a limited number of published works for linear parameter-varying (LPV) systems.

The LPV framework has been introduced to take advantage of the simplicity of LTI control synthesis methods, and simultaneously, accurately capture the dynamics of nonlinear systems over a large operating regime [15]. Despite significant advancements on LPV systems analysis and control synthesis, only limited work on LPV event-triggered control design is available [16–18]. A controller designed for LPV systems quite often depends on the scheduling variables, and this results in some difficulties in the context of LPV event-triggering control compared to LTI systems. In fact, due to this dependency, the scheduling variables must be also sent to the LPV controller and therefore, unlike LTI systems, the scheduling variables and the system states should be considered in the event condition.

In [16], the co-design problem of the control law and the event detector has been studied, where it is assumed that the scheduling variables are not exactly known and their estimated values satisfy a known uncertainty level, which leads to some conservatism. In [17], the co-design problem of event-triggered condition and H_∞ controller is proposed for discrete-time LPV systems. However, in this work, it is assumed that the information about the scheduling variables is available for the controller at all times. In [18], the discretization problem and event-based digital controller design for continuous-time LPV systems subjected to a time-varying networked-induced delay has been investigated. The triggering is based on a significant change of the scheduling variables and an upper bound for the triggering time indicated by $T_{\max}$ that is related to the maximum allowable transmission interval. However, variation of the states or system outputs is not considered in the event detection mechanism. Event-triggered control can be employed to address the data transmission for complex networked-control LPV systems such as process systems to minimize the bandwidth and energy consumption.

In this work, a novel event-triggering co-design approach is proposed to cope with the problem of having an LPV controller dependent on the scheduling variables with the main control objective as the step reference tracking. Although tracking control designs are important in many practical applications, there are only a few studies investigating tracking with event-triggered control [19–21], where in [19] a novel approach to event-triggered tracking control in the case of unmeasurable states has been given for LTI systems. Particularly, to the best of authors' knowledge, there is no study of event-based tracking control for LPV systems.

In this paper, we consider the problem of the co-design of a state feedback controller and the event-triggering conditions for the sensor measurements, the scheduling variables and the controller output for discrete-time LPV systems by designing a Lyapunov function based on the input-to-state stability (ISS) concept. The proposed algorithm addresses the event-based reference tracking control problem in the LPV case, such that the closed-loop system response tracks a desired constant reference signal. A preliminary version of this work was presented in [22], where an event-triggering mechanism only at the sensor node is proposed to reduce the communication load in the sensor to controller channel. However, in this work, two event-triggering mechanisms at the sensor node have been considered aiming at reducing the data transmission of states and scheduling variables separately. Moreover, an event-triggering mechanism is employed here to reduce the data transmission of controller outputs at the controller node. The proposed approach of this paper is *experimentally* validated through a laboratory setup of a tank system. In summary, the contributions of this paper are threefold:

1. An event-triggered state-feedback tracking controller design method is proposed for LPV systems for the first time in the literature.
2. Three separate event-triggering mechanisms are established for transmitting the states, the scheduling variables and the controller outputs through a communication network in order to reduce the data exchange over the network.
3. The proposed approach is experimentally validated on a tank system.

The reminder of this paper is organized as follows. Section 2 presents the problem statement. Section 3 provides the main result of the paper on the co-design of the state feedback control law and the event-triggering strategy for a state-based event-triggered reference tracking control. Effectiveness and capabilities of the proposed methodology are experimentally studied in Section 4, which is followed by conclusions in Section 5.

Notation: Let $\mathbb{R}$ ($\mathbb{R}^+$) and $\mathbb{Z}^+$ stand for the set of real numbers (nonnegative real numbers) and the set of nonnegative integers, respectively. The Euclidean norm of $x \in \mathbb{R}^n$ is denoted by $||x||= \sqrt{x^\top x}$. The i^{th} element of a real vector x is denoted by x^i (subscripts are used for denoting discrete time instants). For a symmetric matrix, $P > 0$ ($P \geq 0$) and $P < 0$ ($P \leq 0$) denote positive-definiteness (positive semi-definite matrix) and negative-definiteness (negative semi-definite matrix), respectively. I and 0 denote the identity and the zero matrices of appropriate dimensions, respectively. A function $\beta : \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\beta(0) = 0$, and to class $\mathcal{K}_\infty$ if additionally $\beta(k) \to \infty$ as $k \to \infty$.

2 Problem Statement

Consider a discrete-time LPV system described by the following state-space model

$$\begin{aligned} x(k+1) &= A(\theta_k)x(k) + Bu_c(k), \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

with state $x(k) \in \mathbb{R}^{n_x}$, output $y(k) \in \mathbb{R}^{n_y}$, input $u_c(k) \in \mathbb{R}^{n_u}$, $B \in \mathbb{R}^{n_x \times n_u}, C \in \mathbb{R}^{n_y \times n_x}$ and scheduling variable $\theta_k \in \mathbb{R}^{n_\theta}$. The variable θ_k lies in a compact set $\Theta \subset \mathbb{R}^{n_\theta}$ for all $k \in \mathbb{Z}^+$ described by the vertices $\nu_{\theta_j}, j = 1, \ldots, n$, i.e.,

$$\Theta := \text{Co}\{\nu_{\theta_1}, \ldots, \nu_{\theta_n}\}, \tag{2}$$

where $n = 2^{n_\theta}$ denotes for the number of vertices in the scheduling region and Co denotes a convex hull.

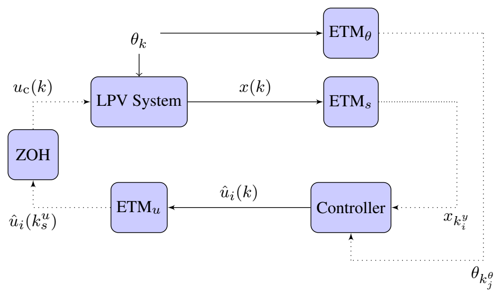

Fig. 1: Event-triggering mechanisms for an LPV system. ETM denotes "event-triggering mechanism".

The LPV state-space representation (1) is considered to be affine in the scheduling variables, i.e.

$$A(\theta_k) = \mathcal{A}_0 + \sum_{l=1}^{n_\theta} \theta_k^l \mathcal{A}_l, \tag{3}$$

where $\mathcal{A}_l$ for $l = 0, \ldots, n_\theta$, are constant matrices and θ_k^l is the l^{th} element of θ_k. This implies that $A(\theta_k)$ can be written in the following polytopic form

$$A(\theta_k) = \sum_{j=1}^{n} \eta_j(\theta_k) A_j, \tag{4}$$

where $\eta_j : \Theta \to \mathbb{R}$ and the mapping $\eta : \Theta \to \mathbb{R}^n$ given by $\eta := [\eta_1 \ldots \eta_n]^\top$ is such that $\eta(\Theta) \in \mathcal{S}$ with

$$\mathcal{S} = \{\mu \in \mathbb{R}^n | \mu_j \geq 0, \forall j \in \{1, \ldots, n\}, \sum_{j=1}^{n} \mu_j = 1\}. \tag{5}$$

Hence, $A(\theta_k)$ lies for each $\theta_k \in \Theta$ in the convex hull $\text{Co}\{A_1, ..., A_n\}$ with n vertices. Affine LPV framework has been extensively used as a core assumption in many LPV control design approaches [23]. As result of using affine LPV models, $A(\theta_k)$ can be written in the polytopic form of (4) which can be used to obtain LMI conditions independent of time-varying scheduling parameters.

The considered problem in this paper is depicted in Figure 1, where ETM$_s$, ETM$_\theta$ and ETM$_u$ denote the event-triggering mechanism on the sensor, scheduling variable and controller side, respectively. It should be noted that although states, scheduling variables and controller outputs may be dependent on each other, for the problem of non-event-based tracking control of an LPV system, the information of states, scheduling variables and controller outputs are sent independently. Therefore, when the problem of event-triggered control for LPV systems is considered, there are separate event-triggering mechanisms for each of states, scheduling variables and controller output signals to determine whether the information of the corresponding signal should be sent through the network. In this setting, three event detectors are implemented to determine time instants k_i^y, k_j^θ, k_s^u $(i, j, s) \in (\mathbb{Z}^+)^3$, where k_i^y, k_j^θ are the time instants at which information of the states $x(k_i^y)$ and the scheduling variables $\hat{\theta}_k = \theta_{k_j^\theta}$ are sent to the controller, respectively, and k_s^u is the time instant at which the controller output $u(k_s^u)$ is sent to the actuator. The main goal of the proposed event-triggered scheme is to reduce the communication rates of the states, scheduling variables and controller signals at both the sensor and the controller outputs.

3 MAIN RESULTS

In this section, our proposed methodology for designing the event-based tracking controller for LPV systems is proposed. The event-triggering conditions along with an LPV state feedback controller are simultaneously designed so that the controlled system output $y(k)$ tracks a constant reference input $r(k)$ and guarantee the tracking performance specification.

Consider a discrete-time controller with an augmented integral action added to eliminate the tracking error, described in state-space form by

$$x_q(k+1) = x_q(k) + (y(k) - r(k)), \tag{6}$$

where $r(k) \in \mathbb{R}^{n_y}$ denotes the reference signal and $x_q(k) \in \mathbb{R}^{n_y}$ corresponds to the state of the integrated tracking error.

The controller signal is chosen as follows

$$u_c(k) = \hat{u}_i(k_s^u), \text{ for } k \in [k_s^u, k_{s+1}^u), \tag{7}$$

where $\hat{u}_i(k)$ denotes the controller output and $u_c(k)$ denotes the signal sent to the actuator with $\hat{u}_i(k) = K_{c_1}(\hat{\theta}_k)x(k_i^y) + K_{c_2}(\hat{\theta}_k)x_q(k_i^y)$ for $k \in [\max(k_i^y, k_j^\theta), \min(k_{i+1}^y, k_{j+1}^\theta))$. By considering (1) and defining the augmented state vector as $\psi(k) = [x^\top(k) \;\; x_q^\top(k)]^\top$, the following augmented LPV system is obtained

$$\psi(k+1) = \bar{A}(\theta_k)\psi(k) + \bar{B}u_c(k) + \bar{E}r(k), \tag{8}$$

where

$$\bar{A}(\theta_k) = \begin{bmatrix} A(\theta_k) & 0 \\ C & I \end{bmatrix}, \; \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \; \bar{E} = \begin{bmatrix} 0 \\ -I \end{bmatrix}.$$

The LPV controller gain $K(\theta_k) = [K_{c_1}(\theta_k) \;\; K_{c_2}(\theta_k)]$ is parametrized in a polytopic form as

$$K(\theta_k) = \sum_{j=1}^{n} \eta_j(\theta_k)K_j. \tag{9}$$

Adding and subtracting the term $\bar{B}\hat{u}_i(k)$ in (8), we obtain

$$\psi(k+1) = \bar{A}(\hat{\theta}_k)\psi(k) + \bar{B}e_u(k) + \bar{B}\hat{u}_i(k) + \bar{E}r(k), \tag{10}$$

where the controller output error is $e_u(k) = u_c(k) - \hat{u}_i(k)$. Define the state measurement error in the interval of $k \in [k_i^y, k_{i+1}^y)$ as $e_\psi(k) = \psi(k_i^y) - \psi(k)$, $\Delta_A(k) = A(\theta_k) - A(\hat{\theta}_k)$ and with $\hat{u}_i(k) = K(\hat{\theta}_k)\psi(k) + K(\hat{\theta}_k)e_\psi(k)$, the augmented system can be represented as follows

$$\begin{aligned}\psi(k+1) = \bar{A}_{\Delta\text{cl}}(\hat{\theta}_k)\psi(k) + \bar{B}K(\hat{\theta}_k)e_\psi(k) \\ + \bar{B}e_u(k) + \bar{E}r(k),\end{aligned} \tag{11}$$

where $\bar{A}_{\Delta\text{cl}}(\hat{\theta}_k) = \bar{A}_\Delta(\hat{\theta}_k) + \bar{B}K(\hat{\theta}_k)$ and $\bar{A}_\Delta(\hat{\theta}_k) = \bar{A}(\hat{\theta}_k) + \bar{\Delta}_A(k)$, with $\bar{\Delta}_A(k) = \text{diag}(\Delta_A(k), 0)$.

Our objective is that whenever the difference between the last transmitted scheduling variable $\hat{\theta}_k$ and the system scheduling variable θ_k reaches a chosen threshold δ_θ, i.e., if there exists an $l \in \{1, \ldots, n_\theta\}$ such that $|e_\theta^l(k)| > \delta_\theta$ with $e_\theta^l(k) = \theta_k^l - \hat{\theta}_k^l$ and $\delta_\theta > 0$, then a new sample of θ_k is transmitted through the network. To choose δ_θ, notice that

$$\Delta_A^\top(k)\Delta_A(k) \leq \bar{\sigma}^2(\Delta_A(k))I, \tag{12}$$

where $\bar{\sigma}$ denotes the maximum singular value. Based on (3), $\Delta_A(k)$ can be written as

$$\Delta_A(k) = \sum_{l=1}^{n_\theta}(\theta_k^l - \hat{\theta}_k^l)\mathcal{A}_l. \tag{13}$$

Based on the intended event-triggering condition on θ_k, it follows that $|\theta_k^l - \hat{\theta}_k^l| \leq \delta_\theta$, $l = 1, ..., n_\theta$, for any $k > 0$. Using the fact that

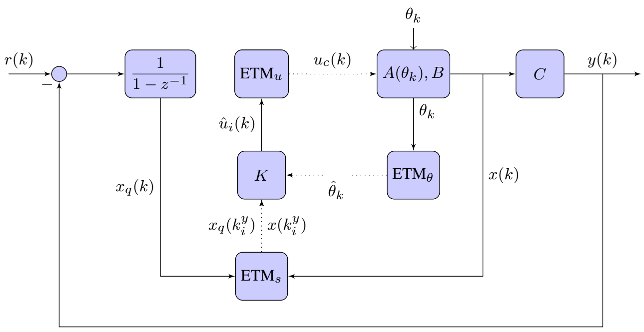

Fig. 2: Event-triggering mechanisms for tracking of a reference signal.

$\bar{\sigma}(\mathcal{X}_1 + \mathcal{X}_2) \leq \bar{\sigma}(\mathcal{X}_1) + \bar{\sigma}(\mathcal{X}_2)$ for any pair of matrices $\mathcal{X}_1$ and $\mathcal{X}_2$, it follows from (13) that

$$\bar{\sigma}(\Delta_A(k)) \leq \delta_\theta \sum_{l=1}^{n_\theta} \bar{\sigma}(\mathcal{A}_l). \tag{14}$$

Hence, (12) implies that

$$\Delta_A^\top(k)\Delta_A(k) \leq \Big(\delta_\theta \sum_{l=1}^{n_\theta} \bar{\sigma}(\mathcal{A}_l)\Big)^2 I. \tag{15}$$

It should be noted that if θ_k is provided continuously to the controller, which means $\delta_\theta = 0$, the uncertainty term $\Delta_A(k)$ becomes zero. The block diagram of the proposed reference tracking methodology is presented in Figure 2.

Remark 1. *In (1), if matrices B and C are dependent on the scheduling variables, we need to replace $B(\theta_k)$ and $C(\theta_k)$ with $B(\hat{\theta}_k)$ and $C(\hat{\theta}_k)$, respectively, to keep the matrix in equation (11) consistent with respect to $\hat{\theta}_k$. Therefore, we have*

$$C(\theta_k) = \Delta_C(k) + C(\hat{\theta}_k),$$
$$B(\theta_k) = \Delta_B(k) + B(\hat{\theta}_k),$$

where $\Delta_C(k) = C(\theta_k) - C(\hat{\theta}_k)$ and $\Delta_B(k) = B(\theta_k) - B(\hat{\theta}_k)$. Then, upper bounds can be obtained for $\Delta_C(k)$ and $\Delta_B(k)$ similarly to the upper bound obtained for $\Delta_A(k)$ in equations (12)-(15).

Next, the main results on the event-triggered controller design problem for the LPV system (1) are given. But, first, the concept of input-to-state stable (ISS) Lyapunov function is reviewed for (11) with $r(k) = 0$.

Theorem 1 (ISS-Lyapunov function [24]). *A function $V : \mathbb{R}^{n_\psi} \times \mathbb{R}^{n_\theta} \to \mathbb{R}^+$ is an ISS-Lyapunov function for (11) ($n_\psi = n_x + n_y$) with $r(k) = 0$ if there exist $\mathcal{K}_\infty$ functions α_1 and α_2 such that for any $\hat{\theta}_k, \hat{\theta}_{k+1} \in \Theta$, and $k \in \mathbb{Z}^+$*

$$\alpha_1(||\psi(k)||) \leq V(\psi(k), \hat{\theta}_k) \leq \alpha_2(||\psi(k)||), \tag{16}$$

and there exists a $\mathcal{K}_\infty$ function ϱ_1 and $\mathcal{K}$ functions γ_1 and γ_2 which satisfy

$$\begin{aligned}V(\psi(k+1), \hat{\theta}_{k+1}) - V(\psi(k), \hat{\theta}_k) \leq -\varrho_1(||\psi(k)||) \\ + \gamma_1(||e_\psi(k)||) + \gamma_2(||e_u(k)||).\end{aligned} \tag{17}$$

Considering a $\mathcal{K}_\infty$ function ϱ_2, it follows that $-\varrho_1(||\psi(k)||) - \varrho_2(||\hat{u}_i(k)||) + \gamma_1(||e_\psi(k)||) + \gamma_2(||e_u(k)||) < -\varrho_1(||\psi(k)||) +$

$\gamma_1(||e_\psi(k)||) + \gamma_2(||e_u(k)||)$, and hence a sufficient condition to satisfy inequality (17) is

$$V(\psi(k+1), \hat{\theta}_{k+1}) - V(\psi(k), \hat{\theta}_k) < -\varrho_1(||\psi(k)||) - \varrho_2(||\hat{u}_i(k)||) + \gamma_1(||e_\psi(k)||) + \gamma_2(||e_u(k)||). \tag{18}$$

Using the inequalities (15) and (18), the system represented by (11) is guaranteed to be asymptotically stable for any $e_\psi(k)$, $e_\theta(k)$ and $e_u(k)$ that satisfy

$$\gamma_1(||e_\psi(k)||) \leq \varrho_1\left(||\psi(k)||\right), \tag{19a}$$

$$|e_\theta(k)| \leq \delta_\theta, \tag{19b}$$

$$\gamma_2(||e_u(k)||) \leq \varrho_2\left(||\hat{u}_i(k)||\right). \tag{19c}$$

Therefore, in order to assure the asymptotic stability of the closed-loop system, the data $\psi(k)$ and θ_k, and the controller output $\hat{u}_i(k)$ should be sent to the controller and to the actuator, respectively, whenever the above inequalities are violated. As a result, the event instants, at which such a violation occurs, are defined iteratively by

$$\begin{aligned} k^y_{i+1} &= \min\{k > k^y_i \mid \gamma_1(||e_\psi(k)||) > \varrho_1(||\psi(k)||)\}, \\ k^u_{s+1} &= \min\{k > k^u_s \mid \gamma_2(||e_u(k)||) > \varrho_2(||\hat{u}_i(k)||)\}, \\ k^\theta_{j+1} &= \min\{k > k^\theta_j \mid |e_\theta(k)| > \delta_\theta\}, \end{aligned} \tag{20}$$

with $k^u_0 = k^y_0 = k^\theta_0 = 0$. The objective is to design the control gain $K(\theta_k)$ such that the augmented closed-loop system (11) is input-to-state stable (ISS) with respect to the signals $e_\psi(k)$ and $e_u(k)$.

Remark 2. *In LTI systems, the event-triggering condition depends on the states of the considered representation of the system and can be usually given as $\gamma_1(||e(k)||) < \varrho_1||x(k)||$, where $e(k) = x(k_i) - x(k)$ at the time instants k_i for $i = 0, 1, 2, ...$; k_i's are the time instants at which the states of the system are sent to the controller. However, dependency of the LPV controller on the scheduling variables makes it difficult to obtain an event-triggering condition independent of the scheduling variables. In fact, the scheduling variables are sent to the LPV controller only at the event time instants. Therefore, for LPV systems, unlike LTI systems, the scheduling variables as well as the states should be considered in the event-triggering condition. In this paper, two separate event-triggering conditions for LPV systems have been presented in (19a) and (19b) to cope with the dependency of the controller on the scheduling variables and to guarantee global asymptotically input to state stability.*

In the following theorem, sufficient LMI conditions to guarantee the stability of the closed-loop system (11) as well as to obtain the parameter-dependent feedback controller gain $K(\theta_k)$.

Theorem 2. *The controlled LPV system described by (7) and (8) with $r(k) = 0$ and $\theta_k \in \Theta$ is asymptotically stable with the event-triggered control input $u_c(k)$ under event conditions $||e_\psi(k)||^2 \leq \sigma_\psi||\psi(k)||^2$, $|e_\theta(k)| \leq \delta_\theta$ and $||e_u(k)||^2 \leq \sigma_u||\hat{u}_i(k)||^2$ if there exist symmetric positive definite matrices $S_i \in \mathbb{R}^{n_\psi \times n_\psi}$, matrices $G_i \in \mathbb{R}^{n_\psi \times n_\psi}$, $F_i \in \mathbb{R}^{r \times n_\psi}$ and positive scalars $\bar{\sigma}_\psi$, ϵ_1 and $\bar{\sigma}_u$ such that the following matrix inequality problem has a feasible solution for $\forall i, j = 1, \ldots, n$*

$$\begin{bmatrix} \mathscr{M}_{11i} & 0 & 0 & G_i\bar{A}_i^\top + F_i^\top\bar{B}^\top & G_i & F_i^\top & G_i \\ \star & \mathscr{M}_{22i} & 0 & F_i^\top\bar{B}^\top & 0 & F_i^\top & 0 \\ \star & \star & I & \bar{B}^\top & 0 & 0 & 0 \\ \star & \star & \star & S_j - \epsilon_1\mathscr{A}_\Delta & 0 & 0 & 0 \\ \star & \star & \star & \star & \bar{\sigma}_\psi I & 0 & 0 \\ \star & \star & \star & \star & \star & \bar{\sigma}_u I & 0 \\ \star & \star & \star & \star & \star & \star & \epsilon_1 I \end{bmatrix} > 0, \tag{21}$$

where $\mathscr{M}_{11i} = G_i + G_i^\top - S_i$ and $\mathscr{M}_{22i} = G_i + G_i^\top - I$ and $\mathscr{A}_\Delta = \mathrm{diag}((\delta_\theta \sum_{l=1}^{n_\theta} \bar{\sigma}(\mathcal{A}_l))^2 I, 0)$. The corresponding vertices of the controller gain (9) defining (7) are $K_i = F_i(G_i^{-1})^\top$, $i = 1, \ldots, n$ and $\sigma_\psi = \bar{\sigma}_\psi^{-1}$ and $\sigma_u = \bar{\sigma}_u^{-1}$.

Proof: Assume that $V(\psi(k), \hat{\theta}_k) = \psi^\top(k)P(\hat{\theta}_k)\psi(k)$, $\varrho_1(||\psi(k)||) = \sigma_\psi\psi^\top(k)\psi(k)$, $\gamma_1(||e_\psi(k)||) = e_\psi^\top(k)e_\psi(k)$, $\varrho_2(||\hat{u}_i(k)||) = \sigma_u\hat{u}_i^\top(k)\hat{u}_i(k)$ and $\gamma_2(||e_u(k)||) = e_u^\top(k)e_u(k)$. Then, the inequality (18) can be written as

$$V(\psi(k+1), \hat{\theta}_{k+1}) - V(\psi(k), \hat{\theta}_k) < -\sigma_\psi\psi^\top(k)\psi(k) + e_\psi^\top(k)e_\psi(k) - \sigma_u\hat{u}_i^\top(k)\hat{u}_i(k) + e_u^\top(k)e_u(k), \tag{22}$$

and it follows from (11) with $r(k) = 0$ that

$$\begin{aligned} &(\bar{A}_{\Delta\mathrm{cl}}(\hat{\theta}_k)\psi(k) + \bar{B}K(\hat{\theta}_k)e_\psi(k) + \bar{B}e_u(k))^\top P(\hat{\theta}_{k+1}) \\ &(\bar{A}_{\Delta\mathrm{cl}}(\hat{\theta}_k)\psi(k) + \bar{B}K(\hat{\theta}_k)e_\psi(k) + \bar{B}e_u(k)) - \\ &\psi^\top(k)P(\hat{\theta}_k)\psi(k) < -\sigma_\psi\psi^\top(k)\psi(k) + e_\psi^\top(k)e_\psi(k) - \\ &\sigma_u\hat{u}_i^\top(k)\hat{u}_i(k) + e_u^\top(k)e_u(k). \end{aligned}$$

By substituting $\hat{u}_i(k) = K(\hat{\theta}_k)\psi(k) + K(\hat{\theta}_k)e_\psi(k)$, one can conclude that

$$\begin{bmatrix} \psi^\top(k) & e_\psi^\top(k) & e_u^\top(k) \end{bmatrix} M(\hat{\theta}_k, \vartheta_k) \begin{bmatrix} \psi(k) \\ e_\psi(k) \\ e_u(k) \end{bmatrix} > 0, \tag{23}$$

where $\vartheta_k = \hat{\theta}_{k+1}$, $\eta(\vartheta_k) \in \mathcal{S}$ in (5), and

$$M(\hat{\theta}_k, \vartheta_k) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ \star & M_{22} & M_{23} \\ \star & \star & M_{33} \end{bmatrix},$$

where

$$\begin{aligned} M_{11} &= P(\hat{\theta}_k) - \sigma_\psi I - \sigma_u K^\top(\hat{\theta}_k)K(\hat{\theta}_k) \\ &\quad - \bar{A}_{\Delta\mathrm{cl}}^\top(\hat{\theta}_k)P(\vartheta_k)\bar{A}_{\Delta\mathrm{cl}}(\hat{\theta}_k), M_{13} = -\bar{A}_{\Delta\mathrm{cl}}^\top(\hat{\theta}_k)P(\vartheta_k)\bar{B}, \\ M_{12} &= -\bar{A}_{\Delta\mathrm{cl}}^\top(\hat{\theta}_k)P(\vartheta_k)\bar{B}K(\hat{\theta}_k) - \sigma_u K^\top(\hat{\theta}_k)K(\hat{\theta}_k), \\ M_{22} &= I - K^\top(\hat{\theta}_k)\bar{B}^\top P(\vartheta_k)\bar{B}K(\hat{\theta}_k) - \sigma_u K^\top(\hat{\theta}_k)K(\hat{\theta}_k), \\ M_{23} &= -K^\top(\hat{\theta}_k)\bar{B}^\top P(\vartheta_k)\bar{B}, \; M_{33} = I - \bar{B}^\top P(\vartheta_k)\bar{B}. \end{aligned}$$

The inequality (23) is equivalent to $M(\hat{\theta}_k, \vartheta_k) > 0$. Hence, by re-arranging the matrix $M(\hat{\theta}_k, \vartheta_k)$ as

$$\begin{bmatrix} P(\hat{\theta}_k) & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} - \begin{bmatrix} \bar{A}_{\Delta\mathrm{cl}}(\hat{\theta}_k) & \bar{B}K(\hat{\theta}_k) & \bar{B} \\ I & 0 & 0 \\ K(\hat{\theta}_k) & K(\hat{\theta}_k) & 0 \end{bmatrix}^\top \begin{bmatrix} P(\vartheta_k) & 0 & 0 \\ 0 & \sigma_\psi I & 0 \\ 0 & 0 & \sigma_u I \end{bmatrix} \begin{bmatrix} \bar{A}_{\Delta\mathrm{cl}}(\hat{\theta}_k) & \bar{B}K(\hat{\theta}_k) & \bar{B} \\ I & 0 & 0 \\ K(\hat{\theta}_k) & K(\hat{\theta}_k) & 0 \end{bmatrix} > 0,$$

which can be rewritten as

$$\begin{bmatrix} P(\hat{\theta}_k) & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} - \Upsilon^\top(\hat{\theta}_k) \begin{bmatrix} P(\vartheta_k) & 0 & 0 \\ 0 & \sigma_\psi I & 0 \\ 0 & 0 & \sigma_u I \end{bmatrix}^{-1} \Upsilon(\hat{\theta}_k) > 0, \tag{24}$$

with

$$\Upsilon(\hat{\theta}_k) = \begin{bmatrix} P(\vartheta_k)\bar{A}_{\Delta\mathrm{cl}}(\hat{\theta}_k) & P(\vartheta_k)\bar{B}K(\hat{\theta}_k) & P(\vartheta_k)\bar{B} \\ \sigma_\psi I & 0 & 0 \\ \sigma_u K(\hat{\theta}_k) & \sigma_u K(\hat{\theta}_k) & 0 \end{bmatrix}, \tag{25}$$

and by using the Schur complement, it can be concluded that

$$\begin{bmatrix} P(\hat\theta_k) & 0 & 0 & \phi_{14}(\hat\theta_k,\vartheta_k) & \sigma_\psi I & \sigma_u K^\top(\hat\theta_k) \\ \star & I & 0 & \phi_{24}(\hat\theta_k,\vartheta_k) & 0 & \sigma_u K^\top(\hat\theta_k) \\ \star & \star & I & \bar B^\top P(\vartheta_k) & 0 & 0 \\ \star & \star & \star & P(\vartheta_k) & 0 & 0 \\ \star & \star & \star & \star & \sigma_\psi I & 0 \\ \star & \star & \star & \star & \star & \sigma_u I \end{bmatrix} > 0. \tag{26}$$

with $\phi_{14}(\hat\theta_k,\vartheta_k) = \bar A^\top_{\Delta\mathrm{cl}}(\hat\theta_k)P(\vartheta_k)$ and $\phi_{24}(\hat\theta_k,\vartheta_k) = K^\top(\hat\theta_k)$ $\bar B^\top P(\vartheta_k)$. From (4) and (5), the inequality (26) is satisfied if

$$\begin{bmatrix} P_i & 0 & 0 & \bar A^\top_{\Delta\mathrm{cl}_i}P_j & \sigma_\psi I & \sigma_u K_i^\top \\ \star & I & 0 & K_i^\top \bar B^\top P_j & 0 & \sigma_u K_i^\top \\ \star & \star & I & \bar B^\top P_j & 0 & 0 \\ \star & \star & \star & P_j & 0 & 0 \\ \star & \star & \star & \star & \sigma_\psi I & 0 \\ \star & \star & \star & \star & \star & \sigma_u I \end{bmatrix} > 0. \tag{27}$$

Multiplying (27) from left and right by $\mathrm{diag}(G_i, G_i, I, P_j^{-1}, \sigma_\psi^{-1}I,$ $\sigma_u^{-1}I)$, where G_i is an invertible matrix with appropriate dimension and by making a change of variables as $S_i = P_i^{-1}$ and $S_j = P_j^{-1}$, it follows that inequality (27) is satisfied if

$$\begin{bmatrix} G_iS_i^{-1}G_i^\top & 0 & 0 & G_i\bar A^\top_{\Delta\mathrm{cl}i} & G_i & G_iK_i^\top \\ \star & G_iG_i^\top & 0 & G_iK_i^\top \bar B^\top & 0 & G_iK_i^\top \\ \star & \star & I & \bar B^\top & 0 & 0 \\ \star & \star & \star & S_j & 0 & 0 \\ \star & \star & \star & \star & \sigma_\psi^{-1} I & 0 \\ \star & \star & \star & \star & \star & \sigma_u^{-1} I \end{bmatrix} > 0. \tag{28}$$

Since $(S_i^{-1/2}G_i^\top - S_i^{-1/2})^\top(S_i^{-1/2}G_i^\top - S_i^{-1/2}) \geq 0$, it follows that

$$G_iS_i^{-1}G_i^\top \geq G_i + G_i^\top - S_i. \tag{29}$$

Then, using $F_i^\top = G_iK_i^\top$, it follows that the inequality (28) is satisfied if

$$\begin{bmatrix} \mathscr{M}_{11i} & 0 & 0 & G_i\bar A^\top_{\Delta\mathrm{cl}i} & G_i & F_i^\top \\ \star & \mathscr{M}_{22i} & 0 & F_i^\top \bar B^\top & 0 & F_i^\top \\ \star & \star & I & \bar B^\top & 0 & 0 \\ \star & \star & \star & S_j & 0 & 0 \\ \star & \star & \star & \star & \bar\sigma_\psi I & 0 \\ \star & \star & \star & \star & \star & \bar\sigma_u I \end{bmatrix} > 0, \tag{30}$$

where $\mathscr{M}_{11i} = G_i + G_i^\top - S_i$, $\mathscr{M}_{22i} = G_i + G_i^\top - I$, $\bar\sigma_\psi = \sigma_\psi^{-1}$ and $\bar\sigma_u = \sigma_u^{-1}$. From (11) and $\bar A_{\Delta\mathrm{cl}i} = \bar A_{\mathrm{cl}i} + \bar\Delta_A(k)$, the inequality (30) can be written as follows

$$\mathscr{M}_{ij} + M_\Delta^\top M_{Gi} + M_{Gi}^\top M_\Delta > 0, \tag{31}$$

where $\bar A_{cli} = \bar A_i + \bar BK_i$ and

$$\mathscr{M}_{ij} = \begin{bmatrix} \mathscr{M}_{11i} & 0 & 0 & G_i\bar A^\top_{\mathrm{cl}i} & G_i & F_i^\top \\ \star & \mathscr{M}_{22i} & 0 & F_i^\top \bar B^\top & 0 & F_i^\top \\ \star & \star & I & \bar B^\top & 0 & 0 \\ \star & \star & \star & S_j & 0 & 0 \\ \star & \star & \star & \star & \bar\sigma_\psi I & 0 \\ \star & \star & \star & \star & \star & \bar\sigma_u I \end{bmatrix},$$

$$M_\Delta = \begin{bmatrix} 0 & 0 & 0 & \bar\Delta_A(k)^\top & 0 & 0 \end{bmatrix},$$
$$M_{Gi} = \begin{bmatrix} G_i^\top & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since $(\epsilon_1 M_\Delta + M_{Gi})^\top(\epsilon_1 M_\Delta + M_{Gi}) \geq 0$, with a positive scalar ϵ_1, it follows that

$$M_\Delta^\top M_{Gi} + M_{Gi}^\top M_\Delta \geq -\epsilon_1 M_\Delta^\top M_\Delta - \epsilon_1^{-1} M_G^\top M_{Gi}. \tag{32}$$

Therefore, the inequality (31) is satisfied if

$$\mathscr{M}_{ij} - \epsilon_1 M_\Delta^\top M_\Delta - \epsilon_1^{-1} M_G^\top M_{Gi} > 0. \tag{33}$$

From (15) and using the Schur complement, it is easy to show that the above inequality is satisfied if

$$\left[\begin{array}{c|c} \tilde{\mathscr{M}}_{ij} & M_{Gi}^\top \\ \hline \star & \epsilon_1 I \end{array}\right] > 0, \tag{34}$$

where $\mathscr{A}_\Delta = \mathrm{diag}\left(\left(\delta_\theta \sum_{l=1}^{n_\theta} \bar\sigma(\mathcal{A}_l)\right)^2 I, 0\right)$ and

$$\tilde{\mathscr{M}}_{ij} = \begin{bmatrix} \mathscr{M}_{11i} & 0 & 0 & G_i\bar A^\top_{\mathrm{cl}i} & G_i & F_i^\top \\ \star & \mathscr{M}_{22i} & 0 & F_i^\top \bar B^\top & 0 & F_i^\top \\ \star & \star & I & \bar B^\top & 0 & 0 \\ \star & \star & \star & S_j - \epsilon_1\mathscr{A}_\Delta & 0 & 0 \\ \star & \star & \star & \star & \bar\sigma_\psi I & 0 \\ \star & \star & \star & \star & \star & \bar\sigma_u I \end{bmatrix}.$$

Finally, (21) can be directly obtained from (34) and this completes the proof. ∎

To solve the corresponding LPV control design problem, a procedure has been proposed for the co-design problem of a state feedback controller and three event-triggering conditions. Next corollary shows the tracking performance of the proposed event-triggered LPV controller for a constant reference signal.

Corollary 1. *Consider the system (1) with controlled LPV system described by (7) and (8) with the controller (9) and event-triggering conditions (19) obtained from Theorem 1. Then, the output $y(k)$ tracks the step reference signal $r(k)$, if the scheduling variables tend to a steady state value.*

Proof: The fact that the system is asymptotically stable and $(\lim_{k\to\infty} \theta_k \to \theta_{\mathrm{ss}})$ implies that for a constant reference input $r(k) = \bar r$, the integral state $x_q(k)$ converges to a steady-state value, i.e., $\lim_{k\to\infty} x_q(k) = \bar x_q$, and hence, $\lim_{k\to\infty}(y(k) - \bar r) = \lim_{k\to\infty}(x_q(k+1) - x_q(k)) = 0$. ∎

Remark 3. *It is noted that the proposed event-based controller is also robust with respect to constant disturbance inputs. Indeed, the integrator part added to the controller is employed to eliminate the steady-state error due to a constant input disturbance or a reference input command [25].*

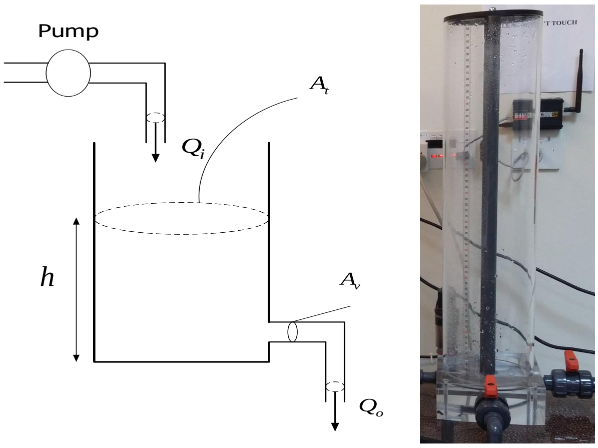

Fig. 3: Schematic and the picture of a single tank in the TTS20 three tank system.

Remark 4. *We note that for a given δ_θ the solution of (21) in Theorem 2 gives an upper bound for the values of σ_ψ and σ_u. These values indirectly affect the performance of the closed-loop system and determine the communication rates. Smaller values of σ_ψ, δ_θ and σ_u for the same controller mean that more data is transmitted to the controller and the actuator, respectively, which results in an improved output tracking performance. The effect of these values is shown in the experimental results in the next section. Moreover, the effect of the difference between the last transmitted scheduling variable and the system scheduling variable (δ_θ), can be seen as an uncertainty in the system. Therefore, for a large value of the predefined threshold the LMI conditions in Theorem 2 likely become infeasible but, on the other hand, less data for scheduling variables are sent to the controller. However, for a very small value of threshold, the effect the uncertainty term is negligible and more data for scheduling variables are sent to the controller.*

4 Experimental Results

To illustrate the efficacy of the proposed event-based control design method, we consider a laboratory setup of a tank system, whose schematic view and picture are shown in Figure 3. The first principles-based dynamic model of the process is given by

$$\dot{h}(t) = -a_z \frac{A_v}{A_t}\sqrt{2gh(t)} + \frac{1}{A_t}Q_i(t), \tag{35}$$

where $h(t)$ is the liquid level, $Q_i(t)$ is the liquid input flow rate, $Q_o(t) = a_z A_v \sqrt{2gh(t)}$ is the liquid output flow rate, A_t and A_v are the surface areas of the cylinder and the connecting pipe, respectively, and a_z is the outflow coefficient varying between 0 and 1. An LPV representation of (35) is given by the following model

$$\dot{h}(t) = A(\theta(t))h(t) + \frac{1}{A_t}Q_i(t), \tag{36}$$

where $\theta(t) = \dfrac{1}{\sqrt{h(t)}}$, $A(\theta(t)) = -a_z \dfrac{A_v}{A_t}\sqrt{2g}\theta(t)$. The parameter values for the setup (TTS20 Three-Tank-System by Gurski-Schramm) are given in Table 1. The highest possible liquid level of the tank is 62cm from the specification of the tank system. The scheduling variable $\theta(t)$ therefore lies in $\Theta := [0.1270, 0.4472]$ for the liquid level of the tank varying between 5cm and 60cm and with the maximum flow rate for the corresponding pump being $100\text{cm}^3/\text{s}$. Using Euler's forward method, the discrete-time approximation of (35) with the sampling period of T_s is obtained as

$$h(k+1) = (1 + T_s A(\theta_k))h(k) + \frac{T_s}{A_t}Q_i(k), \tag{37}$$

where $A(\theta_k) = -a_z \dfrac{A_v}{A_t}\sqrt{2g}\theta_k$. The goal is to track a constant reference level in the tank system via the proposed LPV control design procedure. To reduce the data communication, the information about the current value of the tank level $h(k)$ and the scheduling variable θ_k are sent to a remote controller at time instants $k_i^y, i \in \mathbb{Z}^+$ and $k_j^\theta, j \in \mathbb{Z}^+$, respectively, and the controller output is sent to the pump at time instants $k_s^u, s \in \mathbb{Z}^+$. It is noted that, in this example, the problem of *controller windup* should be considered as a result of the limit on the flow rate for the corresponding pump. In fact, the controller windup occurs because there is a discrepancy between the controller output and the plant input due to the actuator saturation [26]. The LMI feasibility problem of Theorem 2 for $\delta_\theta = 10^{-3}$ is solved with YALMIP toolbox in Matlab and the results for the values of $\sigma_\psi = 5 \times 10^{-4}$, $\sigma_u = 1.2 \times 10^{-3}$ and sampling period $T_s = 0.5$ sec are shown in Figures 4, 5, 6 and 7. We note that here we use the concept of the local truncation error and global error to ensure that sampling time $T_s = 0.5$ sec does not result in divergent of discretization error [27]. In Figure 4, the water level in the tank and its corresponding reference signal are shown under the proposed event-triggered control scheme. As observed from the zoomed portion of Figure 4, the output signal shows some noisy behavior. Level sensor error and small waves on the surface of water in the cylinder are the main sources of this noise. Figures 5, 6 and 7 show the inter-event interval of the event-triggering mechanism for the data transmission from the level sensor and the corresponding scheduling variable to the controller and from the controller to the pump. The value of each stem represents the length of the time period between the current event and the previous one, which illustrates a reduction in data transmission from the level sensor and the scheduling variable to the controller ($\dfrac{\text{the data sent}}{\text{the total samples}}$) to 26.22% and 27.09%, respectively, and from the controller to the pump to 28.76%. The mean-squared error (MSE) of steady state is 1.5×10^{-3}. As it is demonstrated in these figures, the data mostly is sent during the transient response. As a result, the data transmission dramatically decreases when the level of water reaches the desired reference value. In order to investigate the effect of tuning parameters σ_ψ, σ_u and δ_θ, a different scenario is considered for $\sigma_\psi = 5 \times 10^{-5}$, $\delta_\theta = 10^{-4}$ and $\sigma_u = 1.2 \times 10^{-4}$ with the same controller and the same sampling period of $T_s = 0.5$ sec. Figures 8, 9 and 10 illustrate the inter-event interval of the event-triggering mechanism. As observed from these figures, for smaller values of σ_ψ, δ_θ and σ_u, more data

Table 1 Parameter values of the tank system.

Parameter	Value	Unit
A_t	154	cm^2
A_v	0.5	cm^2
a_z	0.45	
g	980.66	cm/sec^2

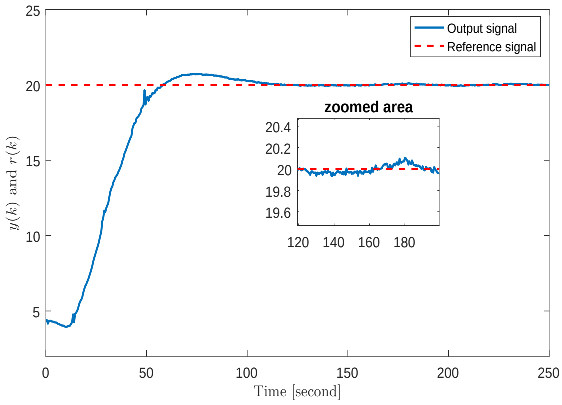

Fig. 4: The output signal $y(k)$ and the reference signal $r(k)$.

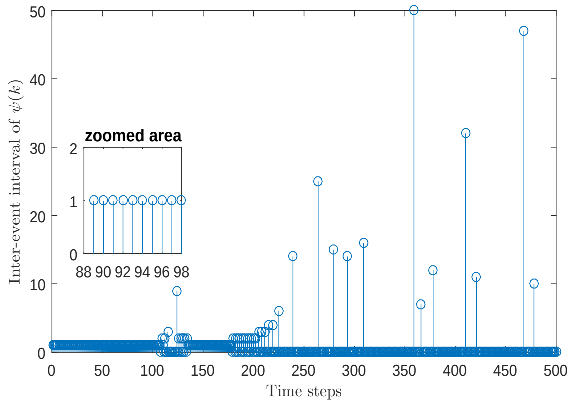

Fig. 5: Inter-event interval of the state event-triggering mechanism for $T_s = 0.5$ sec, $\sigma_\psi = 5 \times 10^{-4}$, $\sigma_u = 1.2 \times 10^{-3}$ and $\delta_\theta = 10^{-3}$.

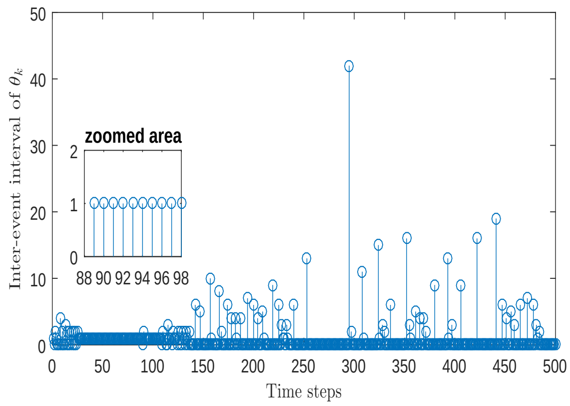

Fig. 6: Inter-event interval of the scheduling variable event-triggering mechanism for $T_s = 0.5$ sec, $\sigma_\psi = 5 \times 10^{-4}$, $\sigma_u = 1.2 \times 10^{-3}$ and $\delta_\theta = 10^{-3}$.

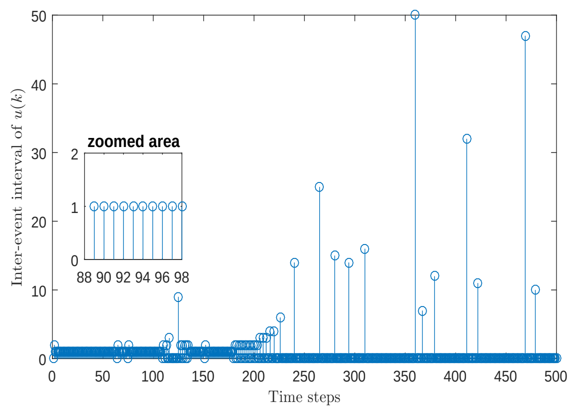

Fig. 7: Inter-event interval of the controller input event-triggering mechanism for $T_s = 0.5$ sec, $\sigma_\psi = 5 \times 10^{-4}$, $\sigma_u = 1.2 \times 10^{-3}$ and $\delta_\theta = 10^{-3}$.

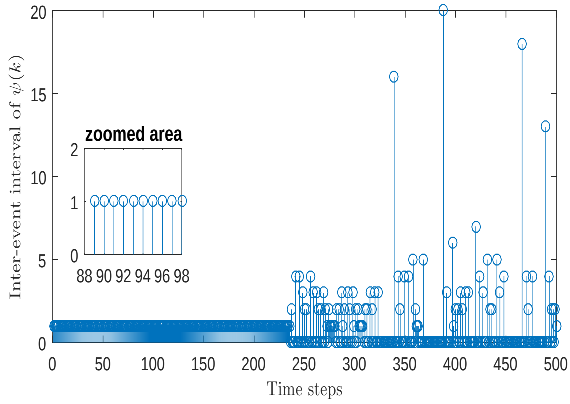

Fig. 8: Inter-event interval of the state event-triggering mechanism for $T_s = 0.5$ sec, $\sigma_\psi = 5 \times 10^{-5}$, $\sigma_u = 1.2 \times 10^{-4}$ and $\delta_\theta = 10^{-4}$.

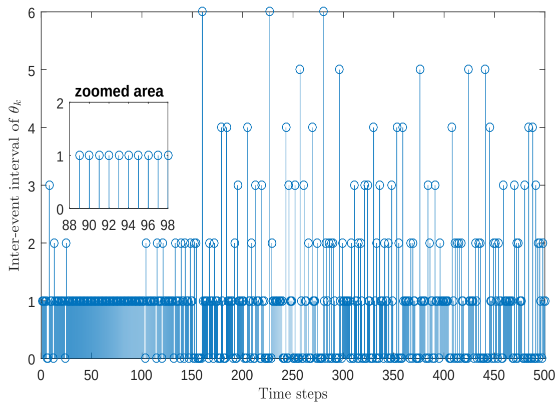

Fig. 9: Inter-event interval of the scheduling variable event-triggering mechanism for $T_s = 0.5$ sec, $\sigma_\psi = 5 \times 10^{-5}$, $\sigma_u = 1.2 \times 10^{-4}$ and $\delta_\theta = 10^{-4}$.

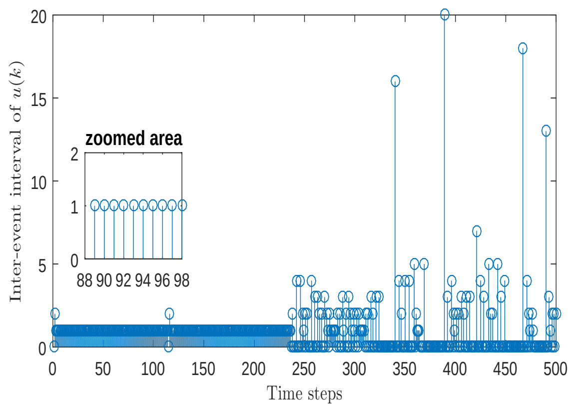

Fig. 10: Inter-event interval of the controller event-triggering mechanism for $T_s = 0.5$ sec, $\sigma_\psi = 5 \times 10^{-5}$, $\sigma_u = 1.2 \times 10^{-4}$ and $\delta_\theta = 10^{-4}$.

are sent from the level and scheduling sensors to the controller and from the controller to the pump. Indeed, the data transmission from the level sensor and scheduling variable to the controller is reduced to 46.69% and 60.55%, respectively, and from the controller to the pump to 47.69%. As a result, the MSE of steady state is 8×10^{-4} which illustrates an improved output tracking performance. To compare the effect of sampling rates, three different sampling periods of $T_s = 0.2, 0.5, 1$ sec are considered to control the level of the liquid in the tank system using the proposed event-based discrete-time LPV controller. For each case, the MSE of steady state is obtained with $\sigma_\psi = 5 \times 10^{-4}$, $\delta_\theta = 10^{-3}$ and $\sigma_u = 1.2 \times 10^{-3}$ which is summarized in Table 2. As observed from the results reported in this

Table 2 MSE with different sampling rates for event-based controller.

Sampling rate (sec.)	MSE	Percentage data sent: sensor to controller	scheduling variable to controller	controller to pump
0.2	1.4×10^{-3}	24.99%	19.49%	25.29%
0.5	1.5×10^{-3}	26.22%	27.09%	25.59%
1	0.1	46.13%	43.89%	43.39%

table, with an increase in the sampling rate, the MSE of the steady state also increases.

5 Conclusion

Although the event-triggered control design problem for LTI systems has been extensively investigated, the mentioned problem for LPV systems, as a class of nonlinear/time-varying systems for which powerful linear control techniques can be exploited, has not been well investigated. In this paper, the event-triggered control design problem for discrete-time LPV systems has been examined. The advantage of applying the event-based LPV control is to reduce the communication rates from the sensors to the controller and from the controller to the actuators. Therefore, two event-triggering mechanisms have been designed at the sensor node aiming to reduce the data transmission of the states and scheduling variables to the controller. Moreover, another event-triggering mechanism has been designed to reduce the data transmission of the controller outputs at the controller. An LMI-based procedure has been developed to design a parameter-dependent state feedback controller along with three event-triggering mechanisms leading to step reference tracking control design for LPV systems in an event-triggered framework.

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