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# Identification of nonlinear process models in an LPV framework

**R. Tóth,* P.M.J. Van den Hof,* J.H.A. Ludlage* and P.S.C. Heuberger***

* *Delft Center for Systems and Control. Delft University of Technology, Mekelweg 2, 2628 CD, Delft, The Netherlands (e-mail: {r.toth,p.m.j.vandenhof, j.h.a.ludlage,p.s.c.heuberger}@tudelft.nl)*

**Abstract:** Driven by the current economical needs, developments in process design and control point out that deliberate operation of chemical process requires better models and control designs than what is offered by the traditional Linear Time-Invariant (LTI) framework. In this paper an identification approach based on Linear Parameter-Varying (LPV) models is introduced for process systems which enables the use of powerful LPV control synthesis tools. LPV systems represent an intermediate step between LTI and nonlinear descriptions as they are capable of describing the system over its whole operating range but preserve many advantages of LTI descriptions. Estimation of LPV models is efficiently solvable by using series expansion type of model structures, like orthonormal basis function models. Advantageous properties of this approach and modeling paradigm are investigated with respect to process models and the added value over LTI models is demonstrated via an example of a continuous stirred tank reactor.

*Keywords:* Nonlinear system identification; linear parameter-varying systems; orthonormal basis functions; nonlinear process control.

## 1. INTRODUCTION

Many chemical processes exhibit nonlinear behavior with a significant contribution to the overall dynamics of the plant. Control of these systems is often found to be challenging. Especially when processes are operated under changing steady state conditions (set-point changes, start-up procedures, grade changes) the nonlinear behavior of the processes becomes apparent, often requiring the use of dedicated nonlinear control approaches. To meet with the increasing performance demands of the chemical industry, often modern control design methods, like model-based control and optimization strategies are applied, e.g. *Nonlinear Model Predictive Control* (NMPC). However, these approaches require accurate dynamic models to obtain satisfactory performance and robustness. For this purpose usually rigorous first-principles models are developed. Important disadvantages of these models are that they suffer from a lack of validation on real-life data, and/or from a high level of model complexity in terms of nonlinear relationships, partial differential terms, etc. It appears to be attractive to identify nonlinear process models from measured data, in order to arrive at relatively simple descriptions of the plant. In this problem the principle question is which (nonlinear) model structures are to be used for the identification.

In nonlinear identification, Hammerstein and Wiener models are widely used, due to their relatively simple structure. Many identification and control methods are available for these model classes. However, such structures can only represent a limited class of nonlinearities and their identification represents a harder problem than in the *Linear Time-Invariant* (LTI) case. Thus, instead of a global nonlinear description of the plant, often an intermediate description is searched for, that preserves the advantageous properties of the LTI models but is still able to represent a wide range of nonlinear systems. Especially in process systems it can often be observed that the process dynamics are well approximated by a linear model, provided that the operating conditions do not change considerably. In order to extend the validity of the linear models over a range of operating conditions, the concept of *Linear Parameter-Varying* (LPV) models appears very attractive (Rugh and Shamma (2000)). As a generalization of the classical concept of gain scheduling, this framework is able to model nonlinear process dynamics in a dedicated modeling framework, where a scheduling variable represents the varying operating conditions of the process. Furthermore, the resulting models are applicable for well-developed extensions of the LTI control strategies, like PID (Kwiatkowski et al. (2009)), MPC (Besselmann et al. (2008)), optimal (Packard (1994)) and robust control (Zhou and Doyle (1998)). In this paper we present a general framework and identification methodology for LPV process models from experimental data. A few examples of existing approaches of LPV identification are Giarré et al. (2006); van Wingerden and Verhaegen (2009); Tóth et al. (2009a,b) and Zhu and Ji (2009).

The basic philosophy that we follow in this paper is to identify LTI models, in several operating points of the process, and to interpolate the resulting models (possibly on the basis of experimental data with varying operating conditions). The resulting global LPV model gives a linear description of the dynamics over the entire operating regime of the plant. This LPV identification method is referred to as the *local approach* and is observed to work well for processes with relatively slow variation of the operating conditions (Tóth (2008)). The choice of model structures to be used for this identification strategy is of crucial importance. The model structure must be easily interpolatable and not affected by the possibly changing system order for different operating conditions. To meet with these requirements we apply series-expansion models based on *Orthonormal Basis Functions* (OBFs). An attractive property of this model structure is that the several local linear models are represented in the same basis, without constraints on possibly changing local model orders, resulting in easily interpolatable model descriptions. A further attractive property of this model structure is the linear-in-the parameters property of the associated one-step-ahead output predictor. The latter property allows the use of simple linear regression algorithms for the identification of these models (being even more attractive in a multivariable setting).

The paper is organized as follows: In Section 2 LPV systems are introduced with the basic model structures offered by this framework. In Section 3 the concept of OBFs is described and their advantages in LPV identification are motivated. In Section 4 the local identification approach of LPV-OBF models is developed and model structure selection is discussed in the proposed setting. In Section 5 the validity of the presented approach is proved through an example of a continuous stirred tank reactor and in Section 6 the conclusions of the paper are presented.

## 2. LPV MODELS

The LPV system class can be seen as an extension of LTI systems as the signal relations are considered to be linear, but the model parameters are assumed to be functions of a time-varying signal, the so-called *scheduling variable* $p : \mathbb{Z} \to \mathbb{P}$ with a *scheduling space* $\mathbb{P} \subseteq \mathbb{R}^n$. This variable is used to indicate the changes in the dynamical signal relations of the plant at different operating conditions. The dynamic description of a LPV system $\mathcal{S}$ can be formalized as a convolution in terms of $p$ and the inputs $u : \mathbb{Z} \to \mathbb{R}^{n_\mathbb{U}}$:

$$y(k) = \sum_{i=0}^{\infty} \mathsf{g}_i(p, k)u(k - i), \tag{1}$$

where $y : \mathbb{Z} \to \mathbb{R}^{n_\mathbb{Y}}$ denotes the output of $\mathcal{S}$ and $k \in \mathbb{Z}$ is the discrete time. The coefficients $\mathsf{g}_i$ of (1) are functions of the scheduling variable and they define the varying linear dynamical relation between $u$ and $y$. This description can also be seen as a series expansion representation of $\mathcal{S}$ in terms of the so called *pulse basis* $\{q^{-i}\}_{i=0}^{\infty}$, where $q$ is the time-shift operator, i.e. $q^{-i}u(k) = u(k - i)$. It can be proven that for an asymptotically stable $\mathcal{S}$, the expansion (1) is convergent (Tóth (2008)).

If the functions $\mathsf{g}_i$ only depend on the instantaneous value of the scheduling signal, i.e. $\mathsf{g}_i(p(k))$, then their functional dependence is called *static*. Otherwise the dependence is called *dynamic*, as the given coefficient not only depends on the instantaneous but also on time-shifted values of $p$. An important property of LPV systems is that for a constant scheduling signal, i.e. $p(k) = \mathsf{p}$ for all $k \in \mathbb{Z}$, (1) is equal to a convolution describing an LTI system as each $\mathsf{g}_i(p, k)$ is constant. Thus, LPV systems can be seen to be similar to LTI systems, but their signal behavior is different due to the variation of the $\mathsf{g}_i$ parameters. Note that in the literature there are many formal definitions of LPV systems, commonly based on particular model structures and parameterizations. The convolution form (1) can be seen as their generalization.

In identification, we aim to estimate a dynamical model of the system based on measured data, which corresponds to the estimation of each $\mathsf{g}_i$ in (1). This estimation is formalized in terms of a model structure, an abstraction of (1), and an identification criterion. A particularly attractive model structure in the LPV case follows by the truncation of (1) to a finite number of expansion terms. Assuming static dependence of $\mathsf{g}_i$, the resulting model reads as

$$y(k) = \sum_{i=0}^{n} \mathsf{g}_i(p(k))u(k - i), \tag{2}$$

which can be seen as the LPV version of the well known LTI *Finite Impulse Response* (FIR) models. Such models have many attractive properties in terms of identification, like linearity-in-the-coefficients that allows to use linear regression for the estimation of the coefficients $\mathsf{g}_i$ if they are linearly parameterized:

$$\mathsf{g}_i(p(k)) = \sum_{j=0}^{n_i} \theta_{ij} f_{ij}(p(k)), \tag{3}$$

where $\theta_{ij} \in \mathbb{R}^{n_\mathbb{Y} \times n_\mathbb{U}}$ are the unknown parameters and $f_{ij}$ are prior selected functions. Furthermore, noise or disturbances in the system can be modeled in an *output error* (OE) sense with this model structure, which allows independent parametrization of the noise model. However, a well known disadvantage of FIR models, both in the LTI and the LPV cases, is that the expansion may have a slow convergence rate, meaning that they require a relatively large number of parameters for an adequate approximation of the system. In order to benefit from the same properties, but achieve faster convergence rate of the expansion, it is attractive to use basis functions which, opposite to $q^{-i}$, have infinite impulse responses. A particular choice of such a basis follows through the use of OBFs.

## 3. ORTHONORMAL BASIS FUNCTIONS

In identification and modeling of LTI systems, the concepts of OBFs based model structures have been extensively studied (Heuberger et al. (2005); Ninness and Gustafsson (1997)). The OBFs are defined as orthonormal transfer functions in $\mathcal{H}_2$ (Hardy space of square integrable complex functions) that form a basis. This way they are able to efficiently represent transfer functions, and hence all associated LTI systems, by their linear combinations.

The transfer function $F \in \mathcal{H}_2^{n_\mathbb{Y} \times n_\mathbb{U}}$ of a (local) LTI model can be written as

$$F(z) = W_0 + \sum_{i=1}^{\infty} W_i \phi_i(z), \tag{4}$$

where $\{\phi_i\}_{i=1}^{\infty}$ is a basis for $\mathcal{H}_2$ and $W_i \in \mathbb{R}^{n_\mathbb{Y} \times n_\mathbb{U}}$. In the theory of *Generalized Orthonormal Basis Functions*, the functions $\phi_i(z)$ can be generated by applying a *Gram-Schmidt orthonormalization* to the sequence of functions

$$\frac{1}{z - \lambda_1}, \; \frac{1}{z - \lambda_2}, \; \ldots \; , \; \frac{1}{z - \lambda_{n_\mathrm{g}}}, \; \frac{1}{(z - \lambda_1)^2}, \; \ldots \tag{5}$$

with stable pole locations $\lambda_1, \ldots, \lambda_{n_\mathrm{g}}$. The choice of these *basis poles* determines the rate of convergence of the series expansion (4). Note that, due to the infinite impulse response characteristics of each $\phi_i(z)$, a faster convergence rate of the expansion can be achieved with (4) than in the FIR case. This construction also provides a way to incorporate prior information about the system in terms of pole locations. For more on OBFs and their properties in the LTI case see Heuberger et al. (2005).

By using a truncated expansion in (4) an attractive model structure for LTI identification results, with a well worked-out theory in terms of variance and bias expressions. The series expansion (4) can be extended to LPV systems, such that for a given basis $\{\phi_i\}_{i=1}^{\infty}$ an LPV system can be written as

$$y(k) = W_0(p,k)u + \sum_{i=1}^{\infty} W_i(p,k)\phi_i(q)u, \qquad (6)$$

where $W_i$ are matrix functions with dynamic dependence on $p$. An obvious choice of model structure is to use a truncated expansion, i.e. truncating (6) to a finite sum in terms of $\{\phi_i\}_{i=1}^{n}$, and to assume static dependence of the coefficients just like in the FIR case (see (2)):

$$y(k) = W_0(p(k))u + \sum_{i=1}^{n} W_i(p(k))\phi_i(q)u, \qquad (7)$$

Note that these expansions are formulated in the time domain (using the shift operator $q$), as there exists no frequency-domain expression for LPV systems. Similar to the FIR case, this structure is linear in the coefficients $\{W_i\}_{i=1}^{n}$. An important question that arises is wether the basis functions $\phi_i$ can be chosen such that a fast rate of convergence can be accomplished for all possible scheduling trajectories $p$.

## 4. IDENTIFICATION OF LPV-OBF MODELS

Next we investigate how LPV-OBF models in the form of (7), i.e. under the assumption of static dependence, can be estimated in practice. Note that, to obtain an LPV-OBF model, first a set of basis functions $\{\phi_i\}_{i=1}^{n}$ must be chosen and subsequently the coefficient functions $\{W_i\}_{i=0}^{n}$ have to be estimated. To simplify the discussion we first assume that the basis functions are given a priori.

*Estimation of the coefficient functions:* LPV-OBF models can be identified based on two approaches. In terms of the so-called "local approach," the LPV model is estimated as a blended model structure based on data collected from the system for $N_{\text{loc}}$ constant scheduling trajectories $p(k) \equiv \mathtt{p}_j \in \mathbb{P}$ (chosen operating points). The resulting *discrete time* (DT) data sequences $\mathcal{D}_j = \{u_j(k), y_j(k)\}_{k=1}^{N_{\text{d}}}$ with $j = 1, \ldots, N_{\text{loc}}$ are recorded for a given sampling time $\mathtt{T}_{\text{d}} > 0$. Based on these data records, samples of the unknown $W_i$ coefficient functions are estimated in (7) for the constant scheduling points $\mathtt{p}_j$. This is accomplished via the estimation of $N_{\text{loc}}$ LTI-OBF models using a standard least-squares criterion in a one-step-ahead prediction error setting with OE noise model. This means that for each $\mathcal{D}_j$, the mean square of the *prediction error*

$$\varepsilon(k) = y_j(k) - \sum_{i=1}^{n_{\text{g}}} \theta_{ij}\phi_i(q)u_j(k), \qquad (8)$$

is minimized during the estimation of the real-valued parameters $\{\theta_{ij}\}$, i.e. samples of $W_i$ at each $\mathtt{p}_j$. In terms of (8) this minimization is a linear regression for which – under the condition that each $\mathcal{D}_j$ is informative – there exists a unique analytical solution. As a second step we use interpolation of each $\{\theta_{ij}\}_{j=1}^{N_{\text{loc}}}$ to obtain estimates of the functions $W_i(p)$, for instance by assuming a polynomial dependence or by making use of splines etc. The strength of the overall approach is that the local estimates can be obtained in closed loop and the well-worked out results of the LTI identification framework can be used. A particular weakness is that transient dynamics of the system for varying $p$ are often poorly modeled. Alternatively, LPV identification can be accomplished in the "global" setting, where (7) is identified based on a data record $\mathcal{D}$ with varying $p$ (see Tóth et al. (2009a,b) for the details).

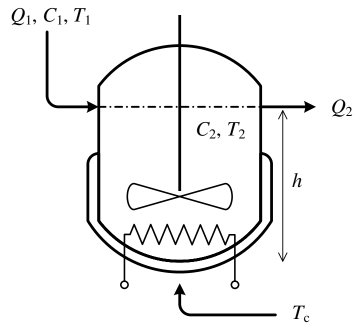


Fig. 1. Continuous stirred tank reactor.

*Choosing the basis functions:* To have an an efficient model structure in terms of (7) with a minimal number of estimated coefficients, a fast convergence rate of (6) is required. This corresponds to an optimal selection of an OBF set $\{\phi_i\}_{i=1}^{n}$ such that the approximation error of (7) is minimal w.r.t. the system. In terms of the local identification, "minimal" corresponds to the span of $\{\phi_i\}_{i=1}^{n}$ having the minimal worst-case representation error (defined via a system norm) for the "local" LTI aspects of the system (at each operating point). In terms of the Kolmogorov theory for OBF models (Oliveira e Silva (1996)), this correspond to the optimization of the pole locations $\lambda_1, \ldots, \lambda_{n_{\text{g}}}$ of the OBFs (see (5)) w.r.t. the set of all possible pole locations associated with the local LTI aspects. In practice, this is accomplished with a so-called *Fuzzy Kolmogorov c-Max* (FKcM) algorithm which, based on samples of the local pole locations (obtained through LTI identification of the system at some operating points), is capable of efficiently solving the optimal OBF selection problem (see Tóth et al. (2009a)).

## 5. EXAMPLE

In order to demonstrate the attractive features of the introduced LPV-OBF identification approach we consider a simulation example of an ideal *Continuous Stirred Tank Reactor* (CSTR) given in Fig. 1. This example describes the chemical conversion, under ideal conditions, of an inflow of substance A to a product B where the corresponding first-order reaction is non-isothermal. For controlling the heat inside the reactor, a heat exchanger with a coolant flow is used. To simplify the problem the following assumptions are taken:

- The liquid in the reactor is ideally mixed.
- The density and the physical properties are constant.
- The liquid level $h$ in the tank is constant, implying that the input and output flows are equal: $Q_1 = Q_2$.

Table 1. Variables & constants associated with the CSTR model and their nominal values.

| | | |
|---|---|---|
| V | Effective volume of the reactor | $5\ \mathrm{m}^3$ |
| $C_1$ | Concentration of the inflow | $800\ \mathrm{kg/m}^3$ |
| $C_2$ | Concentration in the reactor | $213.69\ \mathrm{kg/m}^3$ |
| $Q_1$ | Input flow | $0.01\ \mathrm{m}^3/\mathrm{s}$ |
| $Q_2$ | Output flow | $0.01\ \mathrm{m}^3/\mathrm{s}$ |
| $\mathrm{k}_0$ | Pre-exponential term | 25/s |
| $\mathrm{E_A}$ | Activation energy of the reaction | 30 kJ/kg |
| $T_1$ | Inflow temperature | 428.5 K |
| $T_2$ | The temperature in the reactor | 353 K |
| $T_\mathrm{c}$ | Coolant temperature | 300 K |
| $\rho$ | Density (assumed to be constant) | $800\ \mathrm{kg/m}^3$ |
| $\mathrm{c}_\rho$ | Specific heat | $1\ \mathrm{kJ/kg\cdot K}$ |
| $\Delta H$ | Heat of reaction | 125 kJ/kg |
| $\mathrm{U_{HE}}$ | Heat transfer coefficient | $1\ \mathrm{kJ/kg\cdot s}$ |
| $\mathrm{A_{HE}}$ | Effective surface of the heat exchanger | $1\ \mathrm{m}^2$ |
| $h$ | Level of the liquid in the tank | 5 m |
| R | Gas constant | $8.31\ \mathrm{J/mol\cdot K}$ |

- The reaction is first order with a temperature relation according to the Arrhenius law.
- The shaft work can be neglected.
- The temperature increase of the coolant over the coil can be neglected.

Using the realistic example of a CSTR given in Roffel and Betlem (2007), the variables and constants associated with dynamical behavior of the CSTR are described in Table 1. In this example $Q_1$ and $T_\mathrm{c}$ are used as control signals as they are the typical manipulatable signals used to steer chemical reactors in practice. The control goal is to regulate $T_2$ and $C_2$, so developing a dynamic model that describes the relation between these signals and the input variables is needed in terms of modeling. The nominal values of the variables are given in Table 1, corresponding to the desired steady-state operation of the process.

### 5.1 First-principle modeling

Based on first-principle laws, the following nonlinear differential equations describe the dynamics of the system:

$$\frac{d}{dt}C_2 = \frac{Q_1}{V}(C_1 - C_2) - \mathrm{k}_0 e^{-\frac{\mathrm{E_A}}{\mathrm{R}T_2}} C_2, \tag{9a}$$

$$\frac{d}{dt}T_2 = \frac{Q_1}{V}(T_1 - T_2) - \frac{\mathrm{U_{HE}A_{HE}}}{\rho \mathrm{V c}_\rho}(T_2 - T_\mathrm{c}) + \frac{\Delta H \mathrm{k}_0}{\rho \mathrm{c}_\rho} e^{-\frac{\mathrm{E_A}}{\mathrm{R}T_2}} C_2. \tag{9b}$$

As shown in Roffel and Betlem (2007), if the CSTR corresponding to (9a-b) is operated around the steady state condition given in Table 1, then the system can be well approximated with a 2<sup>nd</sup> order stable LTI model with inputs $u = [\,Q_1\ T_\mathrm{c}\,]^\top$ and outputs $y = [\,C_2\ T_2\,]^\top$. Based on such a model, a PID controller can be designed which ensures disturbance attenuation and provides safe operation of the CSTR around this operating point.

### 5.2 Motivation for LPV models

Assume that the plant where the CSTR is operated receives the raw material (substance A) from different sources. This implies that $C_1$ can have different levels from 50% to 150% of the nominal value. Now apply a 10% step on $Q_1$ at $t = 100$s when the plant is operated in steady state under different $C_1$ levels. The step responses of the CSTR are given in Fig. 2 in terms of the change of $T_2$ and $C_2$ w.r.t. the steady state values (for each $C_1$ level). In the dynamical behavior of $T_2$ and $C_2$ we can observe that both the time constant and relative gain is changing in the responses for different $C_1$ levels. However, the most abrupt changes can be observed in $T_2$ where the relative gain also changes its sign resulting in a non-minimal phase behavior. The latter is a clear evidence that a PID controller designed on the nominal behavior can even destabilize the system if the concentration level of the input flow grows too high. In such scenarios, where the change in the operation conditions causes such a different dynamical behavior, it is important to model the plant for these different scenarios, possibly with a LPV model, which is capable to explain all situations. Therefore, we aim to identify an LPV model of the process which can describe the dynamical behavior of the system w.r.t. to different $C_1$ levels as this seems to be the most important practically relevant problem regarding this application. Furthermore we only intend to model the dynamical relationship between $Q_1$, $T_\mathrm{c}$ and $T_2$, $C_2$ with $C_1$ used as the scheduling variable $p$. Thus all other variables and parameters are assumed to be constant and equal to their nominal values listed in Table 1.

### 5.3 Measurements

To generate realistic measurement records of the system, used for the local identification approach described in Sec. 4, (9a-b) is simulated in continuous-time and DT data records of $C_2$ and $T_2$ are obtained with a sampling period $\mathrm{T_d} = 60$s. This corresponds to an adequate sampling of the transient dynamics with 10 samples during a typical rise time (see Fig. 2). It is also assumed that $Q_1$ and $T_\mathrm{c}$ are manipulated through zero-order-hold actuation synchronized with the sampling period. Simulations are started from the steady state of the process and for excitation *pseudo random binary signals* (PRBS) are injected into $Q_1$ and $T_\mathrm{c}$ at their nominal values with 10% relative amplitude. Note that other excitation signals can also be used to generate informative data sets about the system (see Roffel and Betlem (2007)). To model noise and disturbances related to the measurement of $T_2$ and $C_2$, $e_{T_2}$ and $e_{C_2}$ are added to these signals corresponding to white noise processes with zero mean Gaussian distribution and variance $\sigma_{T_2} = 0.5$, $\sigma_{C_2} = 1.5$. The 3-$\sigma$ levels of $e_{T_2}$ and $e_{C_2}$ are approximately 1% of the nominal values of $T_2$ and $C_2$ with an average *Signal to Noise Ratio* (SNR) of 20dB for $T_2$ and 30dB for $C_2$. Under these conditions, 11 local data records $\mathcal{D}_j$ with length $N_\mathrm{d} = 1000$ are gathered for each $\{400+80j\}_{j=0}^{N_\mathrm{loc}=10}$ level of $C_1$, corresponding to a gridding of the 50% to 150% range. Under the same specifications noiseless data records (with different realization of the PRBS excitation) are also gathered for validation purposes.

### 5.4 Selection of the basis functions

In order to get samples of the possible local pole locations of the system w.r.t. different levels of $C_1$, local DT-LTI models are estimated based on each $\mathcal{D}_j$. For the estimation a 2<sup>nd</sup> order fully parameterized OE model structure with common denominator and no feedthrough term is used and the estimates are calculated with the MATLAB Identification Toolbox. Validation results based on the noiseless data records are computed in terms of the *Best Fit Rate* (BFR) or so-called *fit score*:

$$\text{BFR} := 100\% \cdot \max\left(1 - \frac{\|y-\hat{y}\|_2}{\|y-\bar{y}\|_2}, 0\right), \quad (10)$$

where $\hat{y}$ is the simulated output of the estimated model and $\bar{y}$ is the mean of output $y$ of the CSTR. The achieved rates are given in Fig. 3 with blue $*$, testifying the high validity of these estimated local models. The resulting pole locations of the estimated models are given in Fig. 4 with red $\circ$. On these estimated poles, the FK$c$M algorithm is applied (see Sec. 4) to optimize $n = 5$ OBF functions. These basis functions $\{\phi_i\}_{i=1}^{5}$ will form the model structure in terms of (7) for the LPV identification of the CSTR. The optimized pole locations of the OBFs are given in Fig. 4 with blue $\times$. Performance measures, like the tight best achievable *Kolmogorov bound* (see Tóth et al. (2009a)) given in green in Fig. 4, indicate that these basis have a fast convergence rate, i.e. a negligible representation error in terms of (7), for the dynamics of the CSTR.

### *5.5 LPV identification*

Based on the data records $\mathcal{D}_j$ and the obtained set of OBFs $\{\phi_i\}_{i=1}^{5}$, local samples $\{\theta_{ij}\}_{i=1,\dots,5}^{j=1,\dots,10}$ of the expansion coefficient functions $W_i$ are estimated via linear regression in terms of (8). It is well known that LTI models can only explain the change of $T_2$ and $C_2$ w.r.t. the steady state values of these variables at each $C_1$ due the fact that they correspond to the linearization of (9a-b). Thus these steady state values of $T_2$ and $C_2$ were modeled as a constant, i.e. trim value. The local samples of the coefficients $W_i$ and the trim values are interpolated by using a polynomial approach. By investigating the effect of order selection for the polynomial interpolation it has turned out that the minimal required order is 4 while above 8 no improvement on the approximation error can be observed.

### *5.6 Validation*

The validation results of the estimated LPV-OBF models with polynomial interpolation of order 8 and 4 are given in Fig. 3. These validation results are calculated for a fine grid $\{400+8j\}_{j=0}^{N_{\text{loc}}=100}$ for the levels of $C_1$ (10 times larger than used for identification) in order to investigate the quality of the LPV-OBF models between the interpolation points. The BFR values in Fig. 3 prove that the identified LPV-OBF models are valid between the interpolation points and give accurate local descriptions of the nonlinear system on the operating range of the CSTR w.r.t. $C_1$. The resulting polynomial coefficient functions for the $Q_1 \to T_2$ channel in case of the LPV-OBF model with $8^{\text{th}}$ order interpolation are given in Fig. 5.

Validation is also accomplished w.r.t. to a varying trajectory of $C_1$ in order to test how well the model describes the global behavior of the nonlinear plant. The results are given in Fig. 6. It has been observed that the dependence of the $T_2$ and $C_2$ trim values w.r.t. to $C_1$ has a delay of 11 samples for $T_2$ and 23 samples for $C_2$ (dynamic dependence). It is remarkable that the LPV-OBF model obtained via local identification of the system is able to explain the global nonlinear dynamics with a BFR of 97.54%. It is also obvious from Fig. 6, that the error of $T_2$ is dominated by the transient effects caused by the change

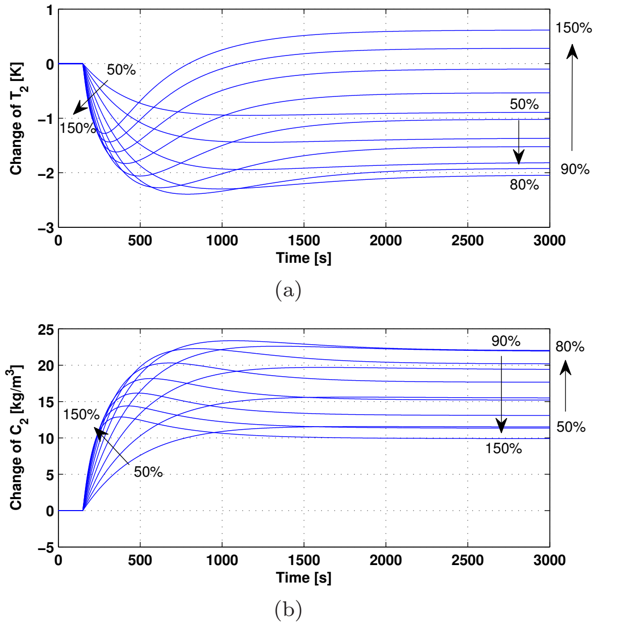


Fig. 2. Change of $T_2$ and $C_2$ when the plant is operated in steady state with 50% $\leftrightarrow$ 150% of nominal $C_1$ and a 10% step is applied on $Q_1$ at $t = 100$s.

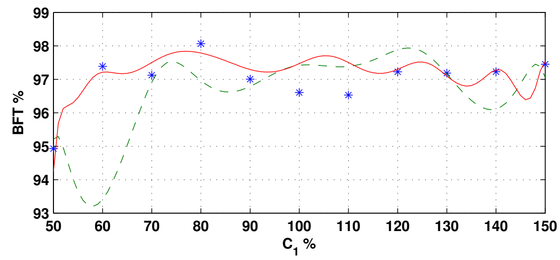


Fig. 3. Validation results of the identified local LTI models, given with blue $*$, together with validation results of the identified LPV-OBF models in terms of BFR computed for a fine grid of $C_1$ levels: $\{400+8\tau\}_{\tau=0}^{100}$. The BFR values are given with red for the LPV-OBF model with $8^{\text{th}}$ order polynomial interpolation, and with a green dashed line for $4^{\text{th}}$ order.

of $C_1$. In case a data record with varying $C_1$ is available for identification, then the transient dynamics caused by the variation of $C_1$ can be easily incorporated into the existing model via a global identification approach (see Tóth et al. (2009b)).

## 6. CONCLUSIONS

This paper demonstrates the strength of an OBFs based LPV identification approach for modeling nonlinear process dynamics. LPV models serve as an intermediate step between rigorous nonlinear process models and simple LTI descriptions commonly used in process control. These models corresponds to a blended structure of a series of LTI models describing the system efficiently over its entire operating regime with powerful control synthesis methods available. The proposed OBF approach gives a well structured way of obtaining LPV models based on local measurements of the system around some operating conditions. The performance of the approach is demonstrated on a simulation example of a CSTR, showing that

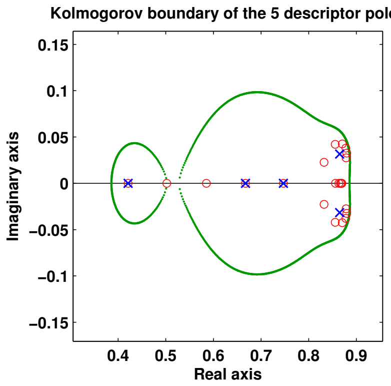


Fig. 4. Optimized OBF poles (blue ×) w.r.t. to the estimated local poles (red ∘). The best achievable Kolmogorov boundary of the optimized OBF poles is given with green.

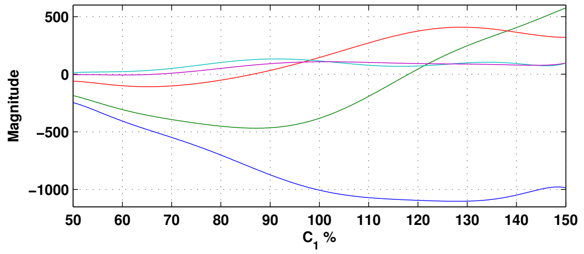


Fig. 5. Coefficient functions of the estimated LPV-OBF model with 8$^{\text{th}}$ order polynomial interpolation for the $Q_1 \to T_2$ channel.

a LPV model that describes the process behavior for different inflow concentrations can be efficiently and cheaply obtained. Such a model can be used to design a controller which can operate the plant for raw ingredients purchased from different sources, providing an efficient and flexible operation of the plant for various production scenarios.


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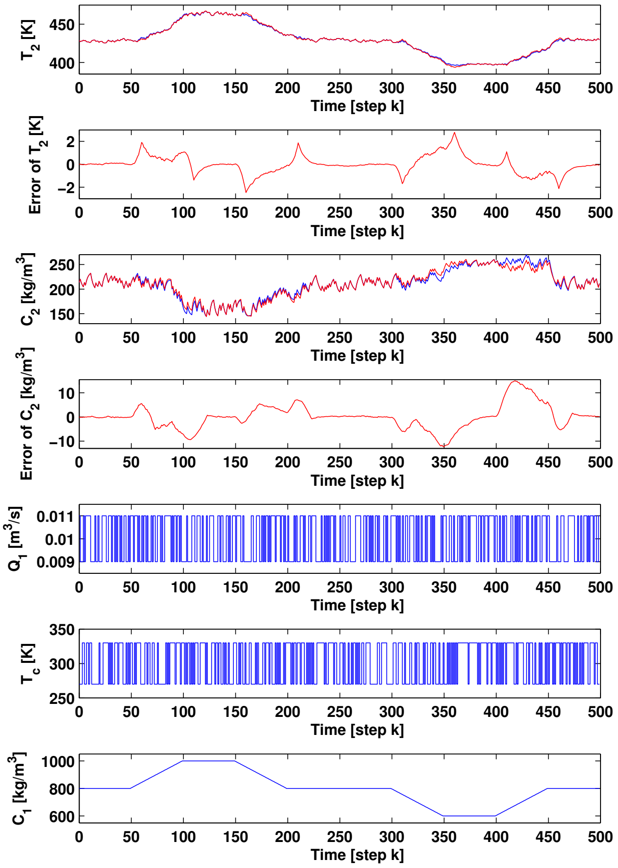


Fig. 6. Validation result of the identified LPV-OBF model with 8$^{\text{th}}$ order polynomial interpolation for varying $C_1$. The response of the true system is given with blue, while the response of the LPV-OBF model is given with red.


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