Abstract—The paper shows the design of a robust control structure for the speed sensorless vector control of the IM, based on the mixed sensitivity (MS) linear parameter variant (LPV) $H_{\infty}$ control theory. The controller makes possible the direct control of the flux and speed of the motor with torque adaptation in noisy environment. The whole control system is tested by intensive simulations and according to the results it shows good dynamic and robust performance. Implementation issues based on a DSP TMS320F243 development platform are also presented.

I. INTRODUCTION

INDUCTION motors (IM) are widely used in the industry due to their simple structure, low cost, and high reliability. Although they are the horsepower of industry, their control is significantly more challenging than that of dc motors, because as a dynamical system they have a highly nonlinear nature with parameter disturbances. This is the reason, why IM’s are still not rival to their dc cousins in a number of high precision applications. Nowadays, therefore, there is a great interest in developing high performance and robust controllers to make induction drives unbeatable in all fields of applications. Especially, these efforts concentrate on controllers that do not need speed sensors to operate, which greatly reduces costs and maintenance. (For details see [6], [11]).

Motivated by this goal, we show the design steps of a robust controller for speed sensorless operation of IMs. The designed system gives the opportunity of fast control of the speed of the motor and the magnetic field associated with the rotor flux ($\Psi_r = [\Psi_{ra}, \Psi_{rb}, \Psi_{rc}]$). This system also possesses the ability to operate in noisy environment and the online adaptation to the load torque ($T_{load}$), which is significant for dynamic tasks. The implemented control law is based on the linear parameter variant (LPV) theory of $H_{\infty}$ control with mixed sensitivity (MS), which has recently appeared in this field [2], [3]. The controller is supported by an I/O linearized reference model and a complex observer synthesized from an extended Kalman filter (EKF) [8], [13] and a $H_{\infty}$ observer [10], [14]. This structure needs only the measurements of the stator currents, and it shows robustness with respect to electrical parameter uncertainties, system and measurement noises. Moreover, the proposed control law is designed to be easy to tune, that holds the possibility of the online tuning of the performance.

The paper is organized as follows. The LPV model of the induction motor is given in Section 2 and the theory of MS LPV $H_{\infty}$ control in Section 3. The design steps of the controller are given in Section 4 and Section 5 includes the simulated results. The implementation with a digital signal processor (DSP) is presented in Section 6 and finally the conclusions are given in Section 7.

II. LPV MODEL OF THE INDUCTION MOTOR

In case of assuming that every variable is continually distributed inside of the machine and the magnetic properties of the rotor are ideal, than the mathematical model of the squirrel-cage IM can be easily derived, if we introduce phasors to describe the density distribution of the electrical quantities and magnetic fields around the stator and the

### TABLE I NOMINAL VALUES OF AN INDUCTION MOTOR

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_s$</td>
<td>lumped stator 3-phase inductance</td>
<td>0.13 H</td>
</tr>
<tr>
<td>$L_r$</td>
<td>lumped rotor 3-p. inductance</td>
<td>0.13 H</td>
</tr>
<tr>
<td>$L_m$</td>
<td>lumped mutual 3-p. inductance</td>
<td>0.12 H</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>leakage factor</td>
<td>0.15</td>
</tr>
<tr>
<td>$R_s$</td>
<td>stator 3-p. resistance</td>
<td>1.86 $\Omega$</td>
</tr>
<tr>
<td>$R_r$</td>
<td>rotor 3-p. resistance</td>
<td>[3Ω, 6Ω]</td>
</tr>
<tr>
<td>$R_0$</td>
<td>$R_0$, at $T_0$ temperature</td>
<td>3Ω</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>rotor angular speed</td>
<td>[-85Hz, 100Hz]</td>
</tr>
<tr>
<td>$\omega_{ref}(t)$</td>
<td>rotor flux angular speed</td>
<td>[-50Hz, 50Hz]</td>
</tr>
<tr>
<td>$T_{load}$</td>
<td>load torque</td>
<td>[-100Nm, 100Nm]</td>
</tr>
<tr>
<td>$m$</td>
<td>rotor winding weight</td>
<td>4 kg</td>
</tr>
<tr>
<td>$c$</td>
<td>specific heat ct. (Al)</td>
<td>0.21</td>
</tr>
<tr>
<td>$T_0$</td>
<td>nominal temperature</td>
<td>18º</td>
</tr>
<tr>
<td>$J$</td>
<td>moment of inertia</td>
<td>0.21 J/kgK</td>
</tr>
<tr>
<td>$p$</td>
<td>number of pole pairs</td>
<td>3</td>
</tr>
<tr>
<td>$F$</td>
<td>fraction coefficient</td>
<td>0.001</td>
</tr>
<tr>
<td>$K_s$</td>
<td>linear heat convection</td>
<td>3.5</td>
</tr>
</tbody>
</table>

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Dénes Fodor, Member, IEEE and Roland Tóth, Student Member, IEEE.
rotor [7], [12]. Based on the phasor theory, the relationship between the flux density, describing the magnetic field, the stator current \( (i_e = [i_{sd}, i_{sq}]^T) \), and the stator voltage \( (u_e = [u_{sd}, u_{sq}]^T) \) can be realized through 2 differential and 2 algebraic equations where the rotor angular speed \( (\omega) \) and the uncertainty of the rotor resistance \( (R_r) \) introduce nonlinearity into the system. From these equations, system (1) follows. This is called the stator oriented \((\alpha, \beta)\) model of the IM, without the motion equation.

\[
\frac{d}{dt} \begin{bmatrix}
\psi_{\alpha, \beta} \\
\psi_{\alpha, \beta}
\end{bmatrix} =
\begin{bmatrix}
 a_1 & -\omega & a_2 \\
\omega & a_1 & 0 & a_2 \\
-\omega a_4 & a_1 & 0 & a_2 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\psi_{\alpha, \beta} \\
i_{sd} \\
i_{sq} \\
i_{ad}
\end{bmatrix}
+ \begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4
\end{bmatrix}
\begin{bmatrix}
 i_{ad} \\
 i_{sd} \\
i_{sq}
\end{bmatrix},
\]  

(1)

where the parameters are defined as follows: \( a_1 = -1 / \tau_r \), \( a_2 = L_m / \tau_r \), \( a_3 = \tau / \sigma \tau_s \), \( a_4 = \tau / \sigma \), \( a_5 = - (\lambda \tau_s + \tau_e) / \sigma \), \( b_1 = 1 / (\sigma \cdot \lambda) \), \( \sigma = 1 - (L_m)^2 / (L_s \cdot L_r) \), \( \lambda = (L_m)^2 / L_s \), \( \tau = L_m / (L_s \cdot L_r) \), \( \tau_e = L_e / L_s \), and \( \tau_s = L_s / L_r \) and their nominal values are given in Table 1.

If \( R_r \) is approximated with equation (2) based on the theory of heating materials (aluminum) with linear convection \((K_r)\) of heat [15]:

\[
\frac{dR_r}{dt} = R_r \cdot \frac{0.86 \cdot R_r}{R_{eq}} \cdot (i_{ad})^2 - K_r \cdot (R_r - R_{eq}),
\]  

(2)

where \( R_{eq} = (245 + T_0) \cdot m \cdot c \), then with the rotor field orientation (RFO) of the phasors [12], the LPV model of the IM is the following:

\[
\frac{d}{dt} x = A(p) \cdot x + B \cdot u, 
\]  

(3)

where

\[
A(p) = \begin{bmatrix}
 a_6 p_1 & a_6 p_2 & 0 \\
 a_6 p_1 & - (a_6 p_2 + a_8) & p_3 \\
 a_6 p_1 & - p_3 & - (a_6 p_2 + a_8)
\end{bmatrix},
\]

\[
x = \begin{bmatrix}
\psi_{sd} \\
i_{sd} \\
i_{sq}
\end{bmatrix},
\]

\[
u = \begin{bmatrix}
i_{ad}
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
b_1 \\
b_2
\end{bmatrix},
\]

\[
A_6 = -1 / L_r, a_7 = L_m / L_r, a_8 = a_10 / \sigma, a_9 = \tau_s / \sigma, a_10 = - \lambda / \sigma.
\]

It is clear that, system (3) is an input affine representation where only the state matrix \( A \) is dependent on the parameters bounded on the polytopic set given in Table 1. These parameters are defined as follows: \( \omega \) of the IM is given by the dynamic motion equation (5) of the rotor.

\[
\frac{d\omega}{dt} = \frac{3 p_1 L_m}{2 J_r} (\psi_{sd} I_{sf} - \frac{p}{J} (T_{es} + F \omega)).
\]  

(5)

\( R_r \) is given by (2) and

\[
\omega_{\text{ref}} = \omega \cdot (L_r R_r i_{sf} / (L_r \psi_{sd})).
\]  

(6)

It is important to note that this RFO LPV model of the IM, gives the possibility to independently control the flux with \( i_{sd} \) (see (3)) and \( \omega \) with \( i_{sq} \) (see (5)). This principle is the cornerstone of the proposed algorithm.

III. MIXED SENSITIVITY LPV H_\infty CONTROL

A. The H_\infty Theory

From the germain works of Zames [5] to the highly improved theories of the MS MIMO controls [9], [14], the H_\infty theory has conquered great portion of today’s controller designs with lots of implemented examples [1]-[3]. Let us give a brief outline of this theory:

For a general control structure with system \( P \) such as in Fig. 1, we are searching for an optimal, robust, and stabilizing controller \( K \) that minimizes the H_\infty norm of the system:

\[
\|G(P, K)\|_{\infty} = \sup_{\omega \in \Delta \cdot \omega(t), t \in (0, t)} \|z(t)\|_{z(t)}, \quad t \in (0, \infty).
\]  

(7)

This optimization is usually solved by a \( \gamma \)-iteration instead of a direct minimization. In each recursive step of this iteration we are looking for a controller that fulfills (8).

\[
\|G(P, K)\|_{\infty} < \gamma.
\]  

(8)

In practice, (8) is solved based on the Ricatti equations or linear matrix inequalities (LMIs). The next \( \gamma \) is computed from the pervious step until the solution gets close enough to its optimal value. Moreover, it is proved that this algorithm converges and produces a robust controller which is stable and fulfills (8) on the whole frequency spectrum [9].

B. LPV Systems

The LPV systems are such linear systems, where the \( A(.) \ldots D(.) \) matrices in the state-space representation (9), (10) are dependent on a \( p(t) \) n dimensional, real parameter
Further, a LPV system can be imagined as a point by point LTI system moving in an \( n \) dimensional system space. Supposing that the system is affine in \( p \) (Condition #1), so each of the \( A(.) \)…\( D(.) \) matrices can be transformed into a \( X(p) = X_0 + X_1(p) + \ldots + X_n(p) \) form, and the \( p \) vector is bounded (Condition #2), then this system can be described by an \( n \) dimensional cube which can be transformed into a polytope (see Fig. 2 and Fig. 3) existing on a \( 2 \) dimensional system space.

If \( P \) is such an LPV system, than it can be represented by a polytope. For each corner of this polytopic set, such LTI \( H_\infty \) controllers can be computed for a given \( \gamma \), which are the corners of a controller set \( K \). This set is equivalent with a LPV \( H_\infty \) controller which fulfills (8). Based on this method, a LPV \( H_\infty \) controller can be designed for the LPV model of the IM, because (3) fulfills Condition #1 and Condition #2. In practice, the solution of controller is achieved through LMIs.

C. Mixed Sensitivity

By introducing frequency filters (weighting functions) on the I/O signals of the system, not only the model of the IM can be more accurately defined, but the properties of the designed controller can be also directly influenced. It can be showed, that the robust stability, disturbance and noise attenuation, and reference tracking of the whole system can be defined, with the frequency definition of the sensitivity function \( S = (I + GK)^{-1} \), the inverse sensitivity function \( T = I - S \), and the closed loop transfer function \( KS \). For a reference tracking objective, the structure presented on Fig. 4 shall be considered.

For an estimation objective by \( H_\infty \) observers, in a similar manner, a MS structure is given on Fig. 5.

IV. SPEED SENSORLESS CONTROLLER DESIGN

To fulfill the recent requirements for an IM drive control, the controller structure in Fig. 6 has been proposed. This structure provides the independent control of the speed and flux based only on the measurement of the stator currents. The mechanism is briefly as follows: The measured noisy 3-phase stator currents are transformed to their vectorial representation with the Clark transformation [12],
and then they are cleaned from the noise by a complex estimation structure, which is the interconnection of a $H_\infty$ observer and a Kalman filter [8], [13]. Here, the Kalman filter provides the estimation of $\omega$ and $R_r$ from the nonlinear equations of the model (2), (5), and the $H_\infty$ observers provides the stator oriented estimation of the rotor flux $(\{\Psi_{ref}, \Psi_{rd}\})$, which is needed for the RFO. After RFO, the input reference signals, $\Psi_{ref}$, $\omega_{ref}$ are transformed to current references, $i_{srd(err)}$ and $i_{sqr(err)}$, by the help of an I/O linearized model of the IM and the previously calculated $\omega$, $R_r$, $\Psi_{rd}$, $i_{sa}$, and $i_{sq}$. Than, the $H_\infty$ controller gets the deviation $i_{srd(err)}$, $i_{sqr(err)}$ from the current reference and calculates the new voltage phasor, which is realized by the Space Vector Pulse Width Modulation (SV PWM) element that directly controls the triggering impulses of the 3 phase inverter generating the desired value of the stator voltage for the IM.

providing good reference tracking on low frequency deviations and preventing the controller to be unnecessary aggressive beyond the cutting-off frequency. The amplification of the filter is only 1dB in the passing region, which gives the possibility to tune the speed and accuracy of the control by external amplification of the current reference signals. Because we greatly reduced the uncertainty by the estimation of $R_r$, there is no need to choose a dynamic filter for $W_r$. By trial and error $W_r$ was designed to be

$$W_r(s) = \text{diag}(0.8, 0.8).$$

Furthermore, for $W_r$, the following filter was introduced to restrict the speed of reference tracking which prevents the controller to be unstable even to the step like changes of the reference signals.

$$W_r(s) = \text{diag}(15 / (s + 15), 15 / (s + 15)).$$

It has turned out, that almost the half of $L_r / R_0 = 43\text{msec}$ for the time constant of (14) provides good tracking without significant overshoots. It is important to note, that because this structure was designed without an integrator an offset error is expected, which is compensated by external gains. However, in opposite to the common practice, this approach also makes possible the external tuning of the controller without destabilizing the whole system.

The optimization was computed through the Matlab function $\text{hinfs}$ which is part of the LMI toolbox. The resulted controller had 5 states, with two inputs and outputs, and it was described with $2^3 = 8$ LTI corner systems, with

$$\gamma = 0.6247.$$  

This means, that without external amplification of $i_{srd(err)}$ and $i_{sqr(err)}$ the steady state offset error is 62.47%.

The I/O linearization of (3) gives the possibility to transform $\Psi_{ref}$, $\omega_{ref}$ into $i_{srd(ref)}$ and $i_{sqr(ref)}$. If the derivatives of $\Psi_{rd}$ and $\omega$ are chosen to $v_1$, $v_2$ virtual inputs equal to $\Psi_{ref}$, $\omega_{ref}$, then the following algebraic equation system provides the reference computation:

$$i_{srd(ref)} = L_r v_1 / (L_\text{m} R_0) + \Psi_{rd} / L_\text{m},$$

$$i_{sqr(ref)} = r_\text{mech} v_1 / (p \Psi_{rd}) + r_\text{mech} (T_{\text{load}} + F \omega),$$

where $r_\text{mech} = 2J L_r / 3p L_\text{m}$. The (15) and (16) equations can handle the transformation task when $\Psi_{rd} \neq 0$ which only occurs when the system is at zero energy. At this point, any value can be assigned to the flux in equations (15) and (16), because this situation exists only for a very short time, during startup.

The flux orientation is handled through the Park transformation [12], to which the needed flux angle is computed from the real and imaginary part of the estimated flux vector. It is clear that for accurate operation we need a very good estimation of the real flux. This is the main reason,
why such a complex structure is used for the estimation task. Even in noisy environment, the $H_\infty$ observers are capable for this very accurate estimation because of their low-pass property. Thus, for the stator oriented LPV flux model of the IM

$$\frac{d}{dt}[\Psi_{ra} \Psi_{rb}] = \begin{bmatrix} a_R p_2 & -p_1 \\ p_1 & a_R p_2 \end{bmatrix} [\Psi_{ra} \Psi_{rb}] + \begin{bmatrix} a_p p_2 & 0 \\ 0 & a_p p_2 \end{bmatrix} [i_{ra} \ i_{rb}],$$  \tag{17}$$

the MS structure on Fig. 8 was used to calculate an $H_\infty$ observer with the hinfgs function. In this structure the frequency of the nonfiltered deviations was chosen to be greater than 300Hz, so the introduced sensitivity filter was

$$W(s) = diag(300 / (s + 30), 300 / (s + 30)).$$  \tag{18}$$

Because any kind of disturbance can shock the system, $W_w$ was omitted for wide interval of functioning. The resulted observer had a $\gamma = 8.49 \cdot 10^{-5}$. Although, this observer calculates the flux vector, the unknown values of $\Omega$ and $R_r$ are still needed. To obtain them, an EKF is attached to the observer. This EKF based on the (1), (2), (5) nonlinear equations, where $u_{sdo}, u_{sdb}, T_{load}$ are used as known inputs and $i_{sa}, i_{sb}$ are used as the measured outputs of the system. Because of the strong dynamical properties of the resulted model, the prediction phase (see [11]) of this EKF is computed through a 3rd order recursive Adams-Bashforth numerical method [4]. In the correction phase of the EKF only the diagonal elements of $Q$ (expected variance of the system noise) and $R$ (expected variance of the measurement noise) were chosen to be nonzeros. It is not a strict assumption, because there is no significant cross coupling between these noises in the real environment. For this reason: $Q_{ij} = 0$, expect $Q(i_{sa}, i_{sa}) = Q(i_{sb}, i_{sb}) = 0.0117h / (L, \sigma);$ \{0, 1, ..., 6\} $\ni$ $i_j$ and $R_{zz} = 0$, expect $R(i_{sa}, i_{sa}) = R(i_{sb}, i_{sb}) = 13.85, R(\Psi_{ra}, \Psi_{ra}) = R(\Psi_{rb}, \Psi_{rb}) = 0.0205; \{0, 1, ..., 4\} \ni j, z,$

where $h$ is the step size of the numerical algorithm.

The estimation also supported by a torque reference model (19), which calculates the expected load torque from the reference signals, weighted by the filter given with (14), and from the measured currents.

$$T_{load} = \tau_{mech}(\Psi_{ra} i_{sa}) - F_o \omega_{ref}.$$  \tag{19}$$

The whole estimation structure was tuned to be perfectly functioning with only 0.5% of prediction error, while heavy measurement noise (Fig. 9), inverter noise (Fig. 10), and 5% parameter uncertainties was introduced into the Matlab simulations, during the design.

\textbf{V. SIMULATION RESULTS}

The controller was tested in Matlab with the help of the Simulink model of the IM. During a very dynamic task where the load torque changed as in Fig. 13, the reference tracking for speed occurred as in Fig. 11, while the rotor flux was changing as in Fig. 12. Additionally, Fig. 14 and Fig. 15 shows the controller given stator voltages, during this simulation.

By looking to these results, it can be concluded that the controller works well even in rapidly changing load conditions (like at 0.5 msec) and its tracking accuracy and dynamics are good even for large reference steps (like at 3.5 msec). The controller was also tested for robustness. With 5% of parameter variance the maximum tracking error in speed was no more than 6.5%.
VI. IMPLEMENTATION WITH TMS320F243

The proposed controller is under implementation on a Digital Spectrum motion control development kit which is powered by a TMS320F243 DSP. This fixed point DSP processor is capable of 20Mips and the processor board contains 8K word Flash ROM. The processor board directly connects to an inverter interface card which produces the PWM signals for a 300Vp AC capable inverter that powers the IM seen on Table 1. This interface card also contains analog to digital converters (ADC) which are used to get to know the values of the stator currents and a dedicated Space-Vector-PWM calculator circuit which is responsible to directly give the PWM signals to the inverter. The connection of the structure is presented on Fig. 16.

VII. CONCLUSION

In this paper our aim was to show the design steps of a state of the art controller for speed sensorless robust operation of the IM, taking into account the load torque changes without the loss of reference accuracy and effectiveness of the whole drive. It is clearly turned out, that with the use of the MS LPV $H_\infty$ control theory the proposed task can be handled and even implemented on a DSP hardware. However, this structure gives the opportunity of accurate control of the given IM with a parameter variance no more than 5%, its usage would be greatly improved with an online tuning algorithm which is in the focus of our future research.

REFERENCES