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Event-triggered Control for Discrete-time Linear Parameter-Varying Systems

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***Abstract*— This paper examines a new event-triggered control design approach for discrete-time linear parameter-varying (LPV) systems to reduce the data transmission of the scheduling variables and states to the controller. A parameter dependent state-feedback controller and an event-triggering condition are designed jointly to stabilize the closed-loop system. Then, a procedure is proposed to obtain an event-based reference tracking controller for the LPV systems such that the steady-state response of the system output tends to a desired reference signal and the required tracking performance specifications are satisfied. Sufficient conditions for the design of the event-triggered controller are obtained in the form of linear matrix inequalities (LMI). Simulation results are presented to demonstrate the effectiveness of the proposed control design approach.**

***Index Terms*— Linear parameter-varying systems; Event-triggered control; Input-to-state stable; Linear-matrix inequality; Output tracking.**

I. INTRODUCTION

The paradigm of periodic control has been presented as the ubiquitous choice for implementing feedback controller laws on digital platforms. However, when a system is operating desirably, it is less preferred to control it periodically which can lead to unnecessarily use of communication resources. Recently, a new approach called event-triggered control has been proposed where the sensing and actuation are done only when they are necessary to maintain the desired operation. The original idea of event-based control proposed in [1], [2] can be used to reduce the communication among the sensors, the controller and the actuators and to reduce the usage of communication resources significantly compared to periodic control. In [3], an introduction to event and self-triggered control systems is presented and event-triggered control as a reactive approach has been distinguished from self-triggered control as a proactive approach where the next sampling or the actuation instance can be computed in advance. The existing approaches can be divided into two general categories, namely, emulation based [4] and co-design based [5], [6] approaches. In the co-design approach, the feedback law and the event-generator are designed jointly whereas in the emulation-based approach, the controller is designed without considering the event-triggered nature of the control system [7]. In the literature, several different event-triggering mechanisms and control strategies are presented (see, e.g., [7]–[18]). However, most of the prior results on the event-based control have been obtained for linear and nonlinear systems and only a few studies have been done for the linear parameter-varying (LPV) systems.

LPV dynamic systems are able to take advantage of the simplicity of linear time-invariant (LTI) control synthesis methods and, at the same time, accurately capture the dynamics of even nonlinear systems over a large operating regime [19]. Yet, a few studies have been done on event-triggered control of LPV systems [20], [21]. In [20], the co-design problem of the event generator and the controller has been addressed where the scheduling variables are assumed to be not exactly known but their estimates satisfy known uncertainty levels which causes some conservatisms. In [21], an event-triggered H_∞ control is proposed for discrete-time linear parameter-varying systems by jointly designing a mixed event-triggering mechanism. A mixed event-triggering mechanism invokes the events when the norm of the difference between the current value and the last transmitted one is larger than the sum of a proportional threshold times the norm of current value and an additional threshold. Compared to the previous works, a novel event-triggering framework is introduced for LPV systems in this paper. In addition, the problem of tracking control is investigated. It should be noted that there are very few studies concerning the tracking control design in event-triggered control [22]–[24]. Particularly, to the best of our knowledge, there is no study in the event-triggered tracking control of LPV systems, although it is an important aspect for practical applications, e.g., in robotic servo and missile flight path control.

In this study, we consider the problem of the co-design of the event-triggering condition and the state-feedback controller for discrete-time LPV systems by applying an input-to-state stable Lyapunov function. Moreover, the algorithm is developed to address event-based reference tracking control in the LPV case, such that the steady-state response of the output tends to a desired reference signal and meets the required tracking performance. To this aim, based on event-triggering conditions, an event generator sends information on states and scheduling variables simultaneously to the controller only when it is needed.

This paper is organized as follows. The problem statement is presented in Section II. The main results are given in Section III, where first, the co-design of the event-triggering condition and the state-feedback controller to stabilize the closed-loop discrete-time LPV system are presented. In the

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second part of this section, a procedure for reference tracking state-based event-triggered control is addressed. In Section IV, properties and performance of the proposed design approaches are studied through numerical simulations. The concluding remarks are given in Section V.

Notation: In this paper, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{Z}^+$ denote the field of real numbers, the set of nonnegative reals and the set of nonnegative integers, respectively. The i^{th} element of a real vector x is denoted by x^i (subscripts are used for denoting discrete-time dependence). We denote by $||x||= \sqrt{x^\top x}$ the Euclidean norm of $x \in \mathbb{R}^n$. When a matrix P is positive definite (including symmetry), we write $P > 0$. If it is positive semi-definite, we use $P \geq 0$. Similarly, for negative definiteness and negative semi definiteness, we write $P < 0$ and $P \leq 0$. By 0 and I, we denote the zero and the identity matrix of appropriate dimensions. A star $(\star)$ in a matrix indicates a transposed quantity in the symmetric position. A function $\beta : \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{K}$, if it is continuous, strictly increasing and $\beta(0) = 0$ and to class $\mathcal{K}_\infty$, if additionally $\beta(k) \to 0$ as $k \to \infty$.

II. PROBLEM STATEMENT

Consider the discrete-time LPV system in the form of

$$\begin{aligned} x(k+1) &= A(\theta_k)x(k) + B(\theta_k)u(k) + E(\theta_k)\omega(k), \\ y(k) &= C(\theta_k)x(k), \\ z(k) &= C_z(\theta_k)x(k), \end{aligned} \tag{1}$$

with state $x(k) \in \mathbb{R}^{n_x}$, output $y(k) \in \mathbb{R}^{n_y}$, input $u(k) \in \mathbb{R}^{n_u}$, disturbance $\omega(k) \in \mathbb{R}^{n_\omega}$, $\theta_k \in \mathbb{R}^{n_\theta}$ being the vector of scheduling variables and $z(k) \in \mathbb{R}^{n_z}$ is the vector of controlled output signals. The variable θ_k lies in a set $\Theta \subset \mathbb{R}^{n_\theta}$ for all $k \in \mathbb{Z}^+$. All the system matrices $A(\theta_k) \in \mathbb{R}^{n_x \times n_x}, B(\theta_k) \in \mathbb{R}^{n_x \times n_u}, C(\theta_k) \in \mathbb{R}^{n_y \times n_x}, C_z(\theta_k) \in \mathbb{R}^{n_z \times n_x}$ and $E(\theta_k) \in \mathbb{R}^{n_x \times n_\omega}$, are assumed to depend on θ_k and can be written in the polytopic form

$$\begin{bmatrix} A(\theta_k) & B(\theta_k) & E(\theta_k) \\ C(\theta_k) & 0 & 0 \\ C_z(\theta_k) & 0 & 0 \end{bmatrix} = \sum_{i=1}^{n} \eta_i(\theta_k) \begin{bmatrix} A_i & B_i & E_i \\ C_i & 0 & 0 \\ C_{z_i} & 0 & 0 \end{bmatrix}, \tag{2}$$

where $\eta_i : \Theta \to \mathbb{R}$ and the mapping $\eta : \Theta \to \mathbb{R}^n$ given by $\eta := [\eta_1 \; ... \; \eta_n]^\top$ is such that $\eta(\Theta) \in \mathcal{S}$ with

$$\mathcal{S} = \{\mu \in \mathbb{R}^n | \mu_i \geq 0, \; i = 1, ..., n \text{ and } \sum_{i=1}^{n} \mu_i = 1\}. \tag{3}$$

Hence, for instance $A(\theta_k)$ lies for each $\theta_k \in \Theta$ in the convex hull $\text{Co}\{A_1, ..., A_n\}$ with n vertices. Note that, in this paper, we use the short hand $\eta_i(\theta_k) := \eta_i(k)$.

The main concept of the proposed control structure is depicted in Figure 1. In this setting, to control the discrete-time linear parameter-varying plant, event generators are employed to determine time instants $k_i \in \mathbb{Z}^+$ with $(i = 0, 1, 2, ...)$ at which information regarding the state sequences $x(k)$ and the scheduling variables θ_k of the system are transmitted to a remote controller. This is intended to reduce communication between the sensors and the control system.

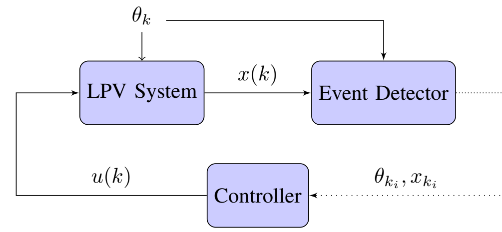

Fig. 1: Event-triggering mechanism for an LPV system.

III. MAIN RESULTS

In this section, first a procedure is proposed to derive an event-triggering condition and a parameter dependent state-feedback controller simultaneously that can stabilize the closed-loop system. Then, the procedure is extended to design the event-based tracking controller for the LPV system represented by (1).

A. State-based event-triggered control

Here, an approach is presented for the problem of the *co-design* of an event generator and a state-feedback controller for the undisturbed discrete-time LPV system (1) (i.e., $\omega(k) \equiv 0$). Consider the parameter-dependent event-triggered controller as

$$u(k) = K(\theta_{k_i})x(k_i), \qquad k \in [k_i, k_{i+1}). \tag{4}$$

Then, the closed-loop system can be written as

$$x(k+1) = A(\theta_k)x(k) + B(\theta_k)K(\theta_{k_i})x(k_i). \tag{5}$$

Define the state measurement error in the interval of $[k_i, k_{i+1})$ as $e(k) = x(k_i) - x(k)$ and the controller gain error as $\Delta K(\theta_{k_i}, \theta_k) = K(\theta_{k_i}) - K(\theta_k)$. The closed-loop system (5) can be written as follows

$$x(k+1) = \mathscr{A}(\theta_k)x(k) + v(k), \tag{6}$$

where $v(k) = B(\theta_k)K(\theta_k)e(k) + B(\theta_k)\Delta K(\theta_{k_i}, \theta_k)x(k_i)$ and $\mathscr{A}(\theta_k) = A(\theta_k) + B(\theta_k)K(\theta_k)$. In the following the main results for designing an event-based controller for LPV systems in the form of (1) is given. However, first, the concept of input-to-state stable (ISS)-Lyapunov function is presented for (6). A function $V : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \to \mathbb{R}^+$ is ISS-Lyapunov function [25] of (6) if there exist $\mathcal{K}_\infty$ functions α_1 and α_2 such that for any $\theta_k, \theta_{k+1} \in \Theta$ and $k \in \mathbb{Z}^+$

$$\alpha_1(||x(k)||) \leq V(x(k), \theta_k) \leq \alpha_2(||x(k)||), \tag{7}$$

and there exists a $\mathcal{K}_\infty$ function α and a $\mathcal{K}$ function γ which satisfy

$$\begin{aligned} V(x(k+1), \theta_{k+1}) - V(x(k), \theta_k) &\leq \\ -\alpha(||x(k)||) &+ \gamma(||v(k)||). \end{aligned} \tag{8}$$

If such an ISS-Lyapunov function exists, then the system represented by (5) is asymptotically stable for any $v(k)$ that satisfies

$$\gamma(||v(k)||) < \alpha(||x(k)||), \qquad \forall k \in \mathbb{Z}^+. \tag{9}$$

This provides the basic idea that the data ($x(k)$ and θ_k) should be sent to the controller whenever the above inequality is violated. The event instants, when such a violation happens, are defined iteratively by

$$k_{i+1} = \min\{k > k_i \mid \gamma(||v(k)||) > \alpha(||x(k)||)\}\,. \quad (10)$$

The goal is to find the parameter dependent controller $K(\theta_k)$ such that the closed-loop system (5) is ISS with respect to signal $v(k)$. The following theorem gives conditions to obtain the parameter dependent state-feedback controller (4) and event-triggering condition (9) to stabilize the closed loop system (6).

Theorem 1. *The closed-loop LPV system resulting from the interconnection depicted in Figure 1 with an LPV plant (1), event triggered controller (4) and event detector (10) is asymptotically stable if there exist symmetric positive definite matrices $S_i \in \mathbb{R}^{n\times n}$, matrices $G_i \in \mathbb{R}^{n_x\times n_x}$, $F_i \in \mathbb{R}^{n_u\times n_x}$ and a positive scalar σ_x such that*

$$\begin{bmatrix} G_i + G_i^\top - S_i & 0 & G_i A_i^\top + F_i^\top B_i^\top & G_i \\ \star & I & I & 0 \\ \star & \star & S_j & 0 \\ \star & \star & \star & \sigma_x I \end{bmatrix} > 0, \quad (11)$$

$$\forall i, j = 1, \ldots, n.$$

Moreover, the controller is given by as $K(\theta_k) = \sum_{i=1}^n \eta_i(k) K_i$ where $K_i = F_i\left(G_i^{-1}\right)^\top$, $i = 1, \ldots, n$ and the event generator law is given as the violation of $||v(k)||^2 \leq \sigma ||x(k)||^2$ where $\sigma = \sigma_x^{-1}$.

Proof. Assume that $V(x(k), \theta_k) = x^\top(k) P(\theta_k) x(k)$, $\alpha(||x(k)||) = \sigma x(k) x^\top(k)$ and $\gamma(||v(k)||) = v^\top(k) v(k)$, then the inequality (8) can be written as

$$\begin{aligned} x^\top(k+1) P(\theta_{k+1}) x(k+1) - x^\top(k) P(\theta_k) x(k) < \\ -\sigma x^\top(k) x(k) + v^\top(k) v(k), \end{aligned} \quad (12)$$

and it follows from (6) that

$$\begin{aligned} &\left(\mathscr{A}(\theta_k) x(k) + v(k)\right)^\top P(\theta_{k+1}) \left(\mathscr{A}(\theta_k) x(k) + v(k)\right) \\ &-x^\top(k) P(\theta_k) x(k) < -\sigma x^\top(k) x(k) + v^\top(k) v(k), \end{aligned}$$

which can be written as

$$\begin{bmatrix} x^\top(k) \; v^\top(k) \end{bmatrix} M(\theta_k, \vartheta_k) \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} > 0, \quad (13)$$

where $\vartheta_k = \theta_{k+1}$ and $\eta(\vartheta_k) \in \mathcal{S}$ in (3) and

$$M(\theta_k, \vartheta_k) = \left[\begin{array}{c|c} -\sigma I + P(\theta_k) & \\ -\mathscr{A}^\top(\theta_k) P(\vartheta_k) \mathscr{A}(\theta_k) & -\mathscr{A}^\top(\theta_k) P(\vartheta_k) \\ \hline \star & -P(\vartheta_k) + I \end{array}\right].$$

The inequality (13) is equivalent to $M(\theta_k, \vartheta_k) > 0$ and hence

$$\begin{aligned} &\begin{bmatrix} P(\theta_k) & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} \mathscr{A}^\top(\theta_k) & I \\ I & 0 \end{bmatrix} \\ &\qquad \begin{bmatrix} P(\vartheta_k) & 0 \\ 0 & \sigma I \end{bmatrix} \begin{bmatrix} \mathscr{A}(\theta_k) & I \\ I & 0 \end{bmatrix} > 0, \end{aligned}$$

which can be rewritten as

$$\begin{aligned} &\begin{bmatrix} P(\theta_k) & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} \mathscr{A}^\top(\theta_k) P(\vartheta_k) & \sigma I \\ P(\vartheta_k) & 0 \end{bmatrix} \begin{bmatrix} P(\vartheta_k) & 0 \\ 0 & \sigma I \end{bmatrix}^{-1} \\ &\qquad \begin{bmatrix} P(\vartheta_k) \mathscr{A}(\theta_k) & P(\vartheta_k) \\ \sigma I & 0 \end{bmatrix} > 0, \end{aligned}$$

and by using Schur complement, it follows that

$$\begin{bmatrix} P(\theta_k) & 0 & \mathscr{A}^\top(\theta_k) P(\vartheta_k) & \sigma I \\ \star & I & P(\vartheta_k) & 0 \\ \star & \star & P(\vartheta_k) & 0 \\ \star & \star & \star & \sigma I \end{bmatrix} > 0. \quad (14)$$

Multiplying (14) from left and right by $\mathrm{diag}\left(G(\theta_k), I, P^{-1}(\vartheta_k), \sigma^{-1}\right)$, where $G(\theta_k)$ is an invertible matrix with appropriate dimension, represented in the polytopic form as

$$G(\theta_k) = \sum_{i=1}^n \eta_i(k) G_i,$$

and by making a change of variables as $S(\theta_k) = P^{-1}(\theta_k)$ and $S(\vartheta_k) = P^{-1}(\vartheta_k)$, it follows that

$$\begin{bmatrix} G(\theta_k) S^{-1}(\theta_k) G^\top(\theta_k) & 0 & G(\theta_k) \mathscr{A}^\top(\theta_k) & G(\theta_k) \\ 0 & I & I & 0 \\ \mathscr{A}(\theta_k) G(\theta_k) & I & S(\vartheta_k) & 0 \\ G^\top(\theta_k) & 0 & 0 & \sigma^{-1} I \end{bmatrix} > 0. \quad (15)$$

Since $\left(S^{-1/2}(\theta_k) G^\top(\theta_k) - S^{-1/2}(\theta_k)\right)^\top \left(S^{-1/2}(\theta_k) G^\top(\theta_k) - S^{-1/2}(\theta_k)\right) \geq 0$, it follows that

$$G(\theta_k) S^{-1}(\theta_k) G^\top(\theta_k) \geq G(\theta_k) + G^\top(\theta_k) - S(\theta_k). \quad (16)$$

Therefore, the inequality (15) is equivalent to

$$\begin{bmatrix} G(\theta_k) + G^\top(\theta_k) - S(\theta_k) & 0 & G(\theta_k) \mathscr{A}^\top(\theta_k) & G(\theta_k) \\ 0 & I & I & 0 \\ \mathscr{A}(\theta_k) G^\top(\theta_k) & I & S(\vartheta_k) & 0 \\ G^\top(\theta_k) & 0 & 0 & \sigma_x I \end{bmatrix} > 0, \quad (17)$$

where $\sigma_x = \sigma^{-1}$. Now, from (2) and (3), the LMI (11) is concluded from (17) and the proof is completed. □

Here, a procedure was proposed for the co-design of a parameter dependent state-feedback controller and an event-triggering condition for the corresponding LPV control problem. In the next section, the procedure is extended for the problem of reference tracking with state-based event-triggered control in the LPV context.

B. Reference tracking with state-based event-triggered control

In this section, a procedure for event-based tracking control of LPV systems is proposed so that the steady-state response of the output $y(k)$ tends to a desired reference signal $r(k)$ and meets the required tracking performance. The schematic of the proposed control structure is shown in Figure 2. There are only a few studies on event-triggered

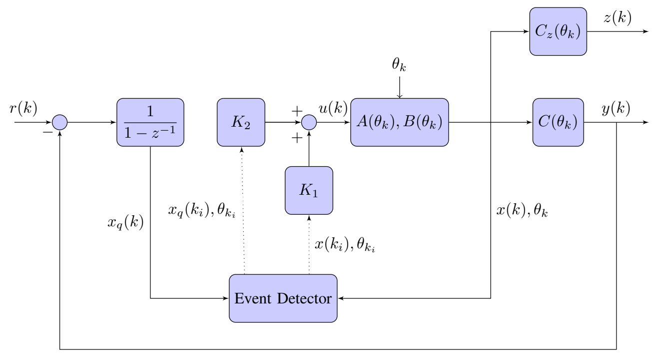

Fig. 2: Event-triggering mechanism for tracking of a reference signal.

tracking control design and specially, to the best of our knowledge, this problem has never been considered for the LPV case. Here, a discrete-time compensator is used for the tracking where the integral action of the tracking error can be represented as follows

$$\begin{aligned} x_q(k+1) &= x_q(k) + (y(k) - r(k)), \\ y(k) &= C(\theta_k)x(k), \end{aligned} \tag{18}$$

where $r(k)$ is the reference signal and $x_q(k)$ is the integrator state. By defining the augmented vector $\psi(k) = \left[x^\top(k) \;\; x_q^\top(k)\right]^\top$ and considering (1), the following augmented LPV system representation is obtained

$$\begin{aligned} \psi(k+1) &= \breve{A}(\theta_k)\psi(k) + \breve{B}(\theta_k)u(k) + \breve{E}(\theta_k)\varpi(k), \\ z(k) &= \breve{C}_z(\theta_k)\psi(k), \end{aligned} \tag{19}$$

where $\varpi(k) = \left[\omega^\top(k) \;\; r^\top(k)\right]^\top$, and

$$\breve{A}(\theta_k) = \begin{bmatrix} A(\theta_k) & 0 \\ C(\theta_k) & I \end{bmatrix}, \quad \breve{B}(\theta_k) = \begin{bmatrix} B(\theta_k) \\ 0 \end{bmatrix}, \tag{20}$$
$$\breve{E}(\theta_k) = \begin{bmatrix} E(\theta_k) & 0 \\ 0 & -I \end{bmatrix}, \quad \breve{C}_z(\theta_k) = \begin{bmatrix} C_z(\theta_k) & 0 \end{bmatrix}.$$

The objective is to design an event-triggered controller such that the measured output $y(k)$ tracks a reference signal $r(k)$ and the effect of the disturbance $\varpi(k)$ is attenuated on the regulated output $z(k)$. So, to ensure the induced ℓ_2-gain performance relating $\varpi(k)$ to regulated output $z(k)$, the following inequality needs to be satisfied

$$\sum_{k=0}^{\infty} z^\top(k)z(k) < \gamma^2 \sum_{k=0}^{\infty} \varpi^\top(k)\varpi(k), \tag{21}$$

where γ is the level of attenuation.

The event-triggered controller is considered in the form of

$$u(k) = \breve{K}(\theta_{k_i})\psi(k_i), \qquad k \in [k_i, k_{i+1}). \tag{22}$$

Therefore, based on (19) and (22), the following representation of the augmented closed-loop system is obtained

$$\begin{aligned} \psi(k+1) &= \breve{A}(\theta_k)\psi(k) + \breve{B}(\theta_k)\breve{K}(\theta_{k_i})\psi(k_i) + \breve{E}(\theta_k)\varpi(k), \\ z(k) &= \breve{C}_z(\theta_k)\psi(k). \end{aligned} \tag{23}$$

Define the state measurement error in the interval of $[k_i, k_{i+1})$ as $e(k) = \psi(k_i) - \psi(k)$ and the controller gain error as $\Delta\breve{K}(\theta_{k_i}, \theta_k) = \breve{K}(\theta_{k_i}) - \breve{K}(\theta_k)$. Therefore, the augmented closed-loop system (23) can be rewritten as follow

$$\begin{aligned} \psi(k+1) &= \breve{\mathscr{A}}(\theta_k)\psi(k) + \breve{E}(\theta_k)\varpi(k) + v(k), \\ z(k) &= \breve{C}_z(\theta_k)\psi(k), \end{aligned} \tag{24}$$

where $v(k) = \breve{B}(\theta_k)\breve{K}(\theta_k)e(k) + \breve{B}(\theta_k)\Delta\breve{K}(\theta_{k_i}, \theta_k)\psi(k_i)$ and $\breve{\mathscr{A}}(\theta_k) = \breve{A}(\theta_k) + \breve{B}(\theta_k)\breve{K}(\theta_k)$. The triggering condition is chosen as

$$||v(k)||^2 \leq \sigma||\psi(k)||^2, \qquad k \in \mathbb{Z}^+, \tag{25}$$

and the triggering times k_i can now be defined as the times when the above inequality is violated.

Again, our objective is to obtain gains $\breve{K}(\theta_{k_i})$ and an event detection parameter σ such that the augmented closed-loop system (23) with $\varpi(k) = 0$ remains ISS with respect to the signal $v(k)$. The following theorem achieves this objective.

Theorem 2. *The output of the LPV system described by (19) with $\theta_k \in \Theta$ for each $k \in \mathbb{Z}^+$ tracks the output reference $r(k)$ and the ℓ_2-gain in (21) is satisfied with the event-triggered controller (22) under event condition $||v(k)||^2 \leq \sigma||\psi(k)||^2$ if there exist symmetric positive definite matrices $S_i \in \mathbb{R}^{n\times n}$, matrices $G_i \in \mathbb{R}^{n\times n}$, $F_i \in \mathbb{R}^{r\times n}$ and positive scalars σ_x and γ such that*

$$\begin{bmatrix} G_i + G_i^\top - S_i & 0 & 0 & G_i\breve{A}_i^\top + F_i^\top\breve{B}_i^\top & G_i & G_i\breve{C}_{zi}^\top \\ \star & I & 0 & I & 0 & 0 \\ \star & \star & \gamma^2 I & \breve{E}_i^\top & 0 & 0 \\ \star & \star & \star & S_j & 0 & 0 \\ \star & \star & \star & \star & \sigma_x I & 0 \\ \star & \star & \star & \star & \star & I \end{bmatrix} > 0, \tag{26}$$
$$\forall i, j = 1, \ldots, n.$$

Moreover, the vertices of the controller gain are given by $\breve{K}_i = F_i\left(G_i^{-1}\right)^\top$, $i = 1, \ldots, n$ and $\sigma = \sigma_x^{-1}$.

Proof. Consider the Lyapunov function as $V(\psi(k), \theta_k) = \psi^\top(k)P(\theta_k)\psi(k)$. To ensure the induced ℓ_2-gain (21) and the ISS condition w.r.t. $v(k)$ the following inequality should be satisfied

$$\begin{aligned} \psi^\top(k+1)P(\theta_{k+1})\psi(k+1) - \psi^\top(k)P(\theta_k)\psi(k) \leq \\ -\sigma\psi^\top(k)\psi(k) - z^\top(k)z(k) + \\ \gamma^2\varpi^\top(k)\varpi(k) + v^\top(k)v(k). \end{aligned} \tag{27}$$

Substituting (24) into (27) gives

$$\begin{aligned} &\Big(\breve{\mathscr{A}}(\theta_k)\psi(k) + \breve{E}(\theta_k)\varpi(k) + v(k)\Big)^\top \\ &P(\theta_{k+1})\Big(\breve{\mathscr{A}}(\theta_k)\psi(k) + \breve{E}(\theta_k)\varpi(k) + v(k)\Big) - \\ &\psi^\top(k)P(\theta_k)\psi(k) < -\sigma\psi^\top(k)\psi(k) - \\ &\psi^\top(k)\breve{C}_z^{\;\top}(\theta_k)\breve{C}_z(\theta_k)\psi(k) + \gamma^2\varpi^\top(k)\varpi(k) + v^\top(k)v(k), \end{aligned}$$

which can be written as

$$\left[\, \psi^\top(k)\;\; v^\top(k)\;\; \varpi^\top(k) \,\right] M(\theta_k, \vartheta_k) \begin{bmatrix} \psi(k) \\ v(k) \\ \varpi(k) \end{bmatrix} > 0, \quad (28)$$

where $\vartheta_k = \theta_{k+1}$ and $\eta(\vartheta_k) \in \mathcal{S}$ in (3) and

$$M(\theta_k, \vartheta_k) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ \star & M_{22} & M_{23} \\ \star & \star & M_{33} \end{bmatrix}, \quad (29)$$

where

$$\begin{aligned} M_{11} &= P(\theta_k) - \sigma I - \breve{\mathscr{A}}^\top(\theta_k) P(\vartheta_k) \breve{\mathscr{A}}(\theta_k) \\ &\quad - \breve{C}_z^\top(\theta_k) \breve{C}_z(\theta_k), \\ M_{12} &= -\mathscr{A}^\top(\theta_k) P(\vartheta_k), \\ M_{13} &= -\breve{\mathscr{A}}^\top(\theta_k) P(\vartheta_k) \breve{E}(\theta_k), \\ M_{22} &= -P(\vartheta_k) + I, \\ M_{23} &= -P(\vartheta_k) \breve{E}(\theta_k), \\ M_{33} &= \gamma^2 I - \breve{E}^\top(\theta_k) P(\vartheta_k) \breve{E}(\theta_k). \end{aligned}$$

The inequality (28) is equivalent to

$$\begin{bmatrix} P(\theta_k) & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \gamma^2 I \end{bmatrix} - \begin{bmatrix} \breve{\mathscr{A}}^\top(\theta_k) & I & \breve{C}_z^\top(\theta_k) \\ I & 0 & 0 \\ \breve{E}^\top(\theta_k) & 0 & 0 \end{bmatrix} \begin{bmatrix} P(\vartheta_k) & 0 & 0 \\ 0 & \sigma I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \breve{\mathscr{A}}(\theta_k) & I & \breve{E}(\theta_k) \\ I & 0 & 0 \\ \breve{C}_z(\theta_k) & 0 & 0 \end{bmatrix} > 0, \quad (30)$$

which can be written as

$$\begin{bmatrix} P(\theta_k) & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \gamma^2 I \end{bmatrix} - \begin{bmatrix} \breve{\mathscr{A}}^\top(\theta_k) P(\vartheta_k) & \sigma I & \breve{C}_z^\top(\theta_k) \\ P(\vartheta_k) & 0 & 0 \\ \breve{E}^\top(\theta_k) P(\vartheta) & 0 & 0 \end{bmatrix} \begin{bmatrix} P(\vartheta_k) & 0 & 0 \\ 0 & \sigma I & 0 \\ 0 & 0 & I \end{bmatrix}^{-1} \begin{bmatrix} P(\vartheta_k) \breve{\mathscr{A}}(\theta_k) & P(\vartheta_k) & P(\vartheta) \breve{E}(\theta_k) \\ \sigma I & 0 & 0 \\ \breve{C}_z(\theta_k) & 0 & 0 \end{bmatrix} > 0, \quad (31)$$

and by using Schur complement it follows that (30) is satisfied if

$$\begin{bmatrix} P(\theta_k) & 0 & 0 & \breve{\mathscr{A}}^\top(\theta_k) P(\vartheta_k) & \sigma I & \breve{C}_z^\top(\theta_k) \\ \star & I & 0 & P(\vartheta_k) & 0 & 0 \\ \star & \star & \gamma^2 I & \breve{E}^\top(\theta_k) P(\vartheta_k) & 0 & 0 \\ \star & \star & \star & P(\vartheta_k) & 0 & 0 \\ \star & \star & \star & \star & \sigma I & 0 \\ \star & \star & \star & \star & \star & I \end{bmatrix} > 0. \quad (32)$$

Multiplying (32) from left and right by $\operatorname{diag}\left(G(\theta_k), I, I, P^{-1}(\vartheta_k), \sigma^{-1} I, I\right)$, where $G(\theta_k)$ is an invertible matrix with appropriate dimension and can be written in the polytopic form as

$$G(\theta_k) = \sum_{i=1}^{n} \eta_i(k) G_i,$$

and by making a change of variables as $S(\theta_k) = P^{-1}(\theta_k)$ and $S(\vartheta_k) = P^{-1}(\vartheta_k)$, the inequality (32) can be written as

$$\begin{bmatrix} \mathcal{G}_{11} & 0 & 0 & G(\theta_k)\breve{\mathscr{A}}^\top(\theta_k) & G(\theta_k) & G(\theta_k)\breve{C}_z^\top(\theta_k) \\ \star & I & 0 & I & 0 & 0 \\ \star & \star & \gamma^2 I & \breve{E}^\top(\theta_k) & 0 & 0 \\ \star & \star & \star & S(\vartheta_k) & 0 & 0 \\ \star & \star & \star & \star & \sigma^{-1} I & 0 \\ \star & \star & \star & \star & \star & I \end{bmatrix} > 0, \quad (33)$$

where $\mathcal{G}_{11} = G(\theta_k) S^{-1}(\theta_k) G^\top(\theta_k)$. Then, according to (16), the inequality (33) can be written as

$$\begin{bmatrix} \mathcal{S}_{11} & 0 & 0 & G(\theta_k)\breve{\mathscr{A}}^\top(\theta_k) & G(\theta_k) & G(\theta_k)\breve{C}_z^\top(\theta_k) \\ \star & I & 0 & I & 0 & 0 \\ \star & \star & \gamma^2 I & \breve{E}^\top(\theta_k) & 0 & 0 \\ \star & \star & \star & 0 & 0 & 0 \\ \star & \star & \star & \star & \sigma_x I & 0 \\ \star & \star & \star & \star & \star & I \end{bmatrix} > 0, \quad (34)$$

where $\mathcal{S}_{11} = G(\theta_k) + G^\top(\theta_k) - S(\theta_k)$ and $\sigma_x = \sigma^{-1}$. Now, using (2) and (3), inequality (26) can be directly obtained from (34) to synthesize the event-based tracking controller for an LPV system and hence the proof is completed. □

IV. Simulation Results

To illustrate the performance of the proposed method, a numerical example is provided. We consider an LPV system described by the following state-space matrices

$$A(\theta_k) = \begin{bmatrix} 0.02 & 1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 + \theta_k \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix},$$

$$C = \left[\, 1 \quad 0 \quad 0.2 \,\right], \qquad C_z = \left[\, 0.5 \quad 0 \quad 0 \,\right],$$

and $\omega(k) = 0.2 \sin(k), \theta_k \in [0, 0.5], k \in \mathbb{Z}^+$. By minimizing γ and σ_x ($\sigma = \sigma_x^{-1}$) w.r.t. the LMI in Theorem 2, K_1, K_2, γ and σ are obtained as follows

$$\begin{aligned} K_1 &= [-0.64 \;\; -0.08 \;\; 0.13 \;\; 0.62], \\ K_2 &= [-0.65 \;\; -0.08 \;\; 0.20 \;\; 0.63], \\ \gamma &= 4.12, \quad \sigma = 0.05. \end{aligned}$$

If the value of σ is increased, then data is less frequently transmitted. In the other words, for smaller values of σ, better tracking is obtained because more data is sent to the controller. In Figure 3, the output signal $y(k)$ under the proposed event-triggered control scheme and the reference signal $r(k)$ for two values of $\sigma = 0.02, 0.05$ are shown. As it is displayed in this figure, for the smaller value of σ, better tracking is obtained. Figure 4 shows the inter-event interval of the event detector for both values of σ. The value of each stem indicates the length of the time period between that event and the previous one which shows a reduction of data transmission to 59.8% for $\sigma = 0.05$ and to 91.7% for $\sigma = 0.02$. As the figures confirm, more data is sent for smaller σ.

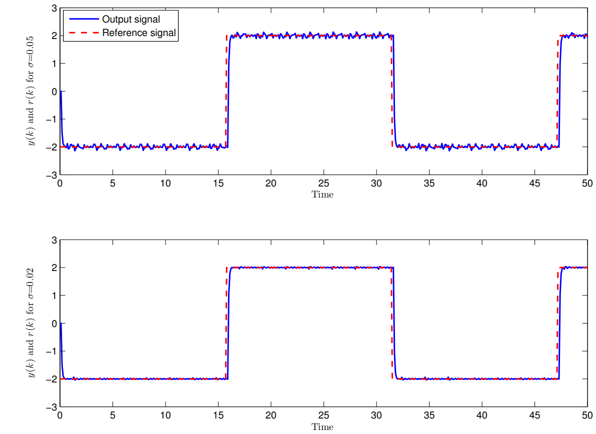

Fig. 3: The output signal $y(k)$ and the reference signal $r(k)$ for $\sigma = 0.05$ and $\sigma = 0.02$.

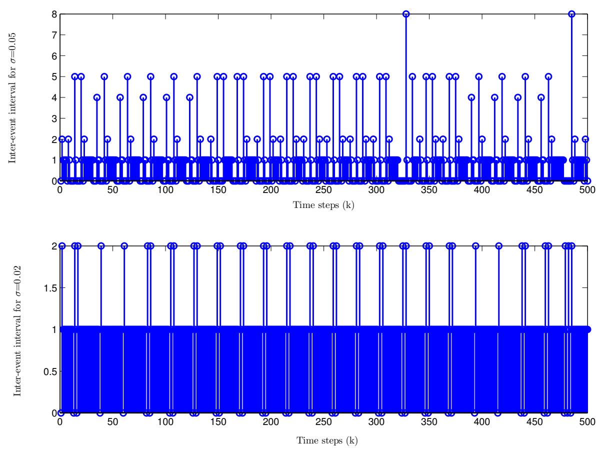

Fig. 4: Inter-event interval of the event detector for $\sigma = 0.05$ and $\sigma = 0.02$.

V. Conclusion

While there has been much work devoted to the event-based control for linear time-invariant systems, LPV systems, as an important class of dynamic systems which have the advantages of linear control techniques has not been well investigated in this field. In this paper, a novel scheme of event-based control for discrete-time LPV systems has been proposed. The advantage of using event-triggered LPV control is to reduce the data transmission (scheduling variables and states information) between the system and the controller. The parameter-dependent state feedback controller and the event-triggering condition have been designed simultaneously to stabilize the resulting closed-loop system. Moreover, the procedure has been extended for output tracking control of LPV systems in an event-triggered scheme.

References

[1] K.-E. Årzén, "A simple event-based PID controller," in *Proc. of the 14th World Congress of IFAC*, Beijing, China, Jan. 1999.

[2] K. J. Aström, "Event based control," in *Analysis and Design of Nonlinear Control Systems*. Springer Berlin Heidelberg, 2008, pp. 127–147.

[3] W. Heemels, K. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *Proc. of the 51st IEEE Conference on Decision and Control*, Dec 2012, pp. 3270–3285.

[4] A. Eqtami, D. Dimarogonas, and K. Kyriakopoulos, "Event-triggered control for discrete-time systems," in *Proc. of the American Control Conference*, June 2010, pp. 4719–4724.

[5] M. Abdelrahim, R. Postoyan, J. Daafouz, and D. Nesic, "Co-design of output feedback laws and event-triggering conditions for linear systems," in *Proc. of the 53rd IEEE Conference on Decision and Control*, Dec 2014, pp. 3560–3565.

[6] X. Meng and T. Chen, "Event detection and control co-design of sampled-data systems," *International Journal of Control*, vol. 87, no. 4, pp. 777–786, 2014.

[7] M. Donkers and W. Heemels, "Output-based event-triggered control with guaranteed L_∞-gain and improved and decentralized event-triggering," *IEEE Transactions on Automatic Control*, vol. 57, no. 6, pp. 1362–1376, June 2012.

[8] J. Almeida, C. Silvestre, and A. Pascoal, "Self-triggered output feedback control of linear plants in the presence of unknown disturbances," *IEEE Transactions on Automatic Control*, vol. 59, no. 11, pp. 3040–3045, Nov 2014.

[9] X. Meng and T. Chen, "Event triggered robust filter design for discrete-time systems," *IET Control Theory Applications*, vol. 8, no. 2, pp. 104–113, January 2014.

[10] W. Heemels, M. Donkers, and A. Teel, "Periodic event-triggered control for linear systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 847–861, April 2013.

[11] D. Borgers and W. Heemels, "Event-separation properties of event-triggered control systems," *IEEE Transactions on Automatic Control*, vol. 59, no. 10, pp. 2644–2656, Oct 2014.

[12] C. Xia and H. Fei, "Stability of event-triggered output-feedback control systems," in *Proc. of the 30th Chinese Control Conference*, July 2011, pp. 1184–1189.

[13] J. Almeida, C. Silvestre, and A. Pascoal, "Observer based self-triggered control of linear plants with unknown disturbances," in *Proc. of the American Control Conference*, June 2012, pp. 5688–5693.

[14] M. Mazo and P. Tabuada, "Decentralized event-triggered control over wireless sensor/actuator networks," *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2456–2461, Oct 2011.

[15] M. Mazo and M. Cao, "Decentralized event-triggered control with asynchronous updates," in *Proc. of the 50th IEEE Conference on Decision and Control*, Dec 2011, pp. 2547–2552.

[16] J. Lunze and D. Lehmann, "A state-feedback approach to event-based control," *Automatica*, vol. 46, no. 1, pp. 211 – 215, 2010.

[17] D. Lehmann and J. Lunze, "Event-based output-feedback control," in *Proc. of the 19th Mediterranean Conference on Control Automation*, June 2011, pp. 982–987.

[18] C. Stöcker and J. Lunze, "Event-based control of nonlinear systems: An input-output linearization approach," in *Proc. of the 50th IEEE Conference on Decision and Control*, Dec 2011, pp. 2541–2546.

[19] J. Shamma, "An overview of LPV systems," in *Control of Linear Parameter Varying Systems with Applications*, J. Mohammadpour and C. W. Scherer, Eds. Springer US, 2012, pp. 3–26.

[20] L. Shanbin and X. Bugong, "Event-triggered control for discrete-time uncertain linear parameter-varying systems," in *Proc. of the 32nd Chinese Control Conference*, July 2013, pp. 273–278.

[21] S. Li, D. Sauter, and B. Xu, "Co-design of event-triggered control for discrete-time linear parameter-varying systems with network-induced delays," *Journal of the Franklin Institute*, vol. 352, no. 5, pp. 1867 – 1892, 2015.

[22] L. Jetto and V. Orsini, "A new event-driven output-based discrete-time control for the sporadic MIMO tracking problem," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 5, pp. 859–875, 2014.

[23] S. Hu, Y. Zhang, and Z. Du, "Network-based H_∞ tracking control with event-triggering sampling scheme," *IET Control Theory Applications*, vol. 6, no. 4, pp. 533–544, March 2012.

[24] P. Tallapragada and N. Chopra, "On event triggered trajectory tracking for control affine nonlinear systems," in *Proc. of the 50th IEEE Conference on Decision and Control*, Dec 2011, pp. 5377–5382.

[25] Z.-P. Jiang and Y. Wang, "Input-to-state stability for discrete-time nonlinear systems," *Automatica*, vol. 37, no. 6, pp. 857 – 869, 2001.